

Math 434: Problem Sheet 5

“By relieving the brain of all unnecessary work, a good notation sets it free to concentrate on more advanced problems, and, in effect, increases the mental power of the race.” – Alfred North Whitehead

0.1 Rings part 1

1. Do **both** of the following.
 - (a) Prove all of the parts of Theorem 12.1 in Gallian.
 - (b) Given a ring R , the set of formal power series $p(t) = a_0 + a_1t + a_2t^2 + \cdots +$ (‘formal’ means there is no requirement of convergence) is a ring. (Denoted $R[[t]]$.) Show that $R[[t]]$ is a ring and prove that a formal power series $p(t)$ is invertible if and only if a_0 is a unit of R .
2. Let Q denote the rational numbers (you may use the fact that Q is a field), $Q[\alpha]$ the smallest subring of C (the complex numbers) containing α , and $Q[\alpha, \beta]$ the smallest subring of C containing both α and β . Let $\alpha = \sqrt{2}$, $\beta = \sqrt{3}$ and $\gamma = \alpha + \beta$. Prove that $Q[\alpha, \beta] = Q[\gamma]$.
3. Prove the distributive law and the cancellation law of addition for the natural numbers. You may assume commutativity and associativity have already been proven.