

January 28, 2002

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

*“Reductio ad absurdum, which Euclid loved so much, is one of a mathematician’s finest weapons. It is a far finer gambit than any chess play: a chess player may offer the sacrifice of a pawn or even a piece, but a mathematician offers the game.”* – Godfrey H. Hardy

**Type I Problems**

1. Determine if the following are tautologies.

- (a)  $p \implies (q \implies p)$
- (b)  $[p \implies (q \implies r)] \implies [q \implies (p \implies r)]$
- (c)  $(p \vee q) \iff (\sim p) \wedge (\sim q)$
- (d)  $p \wedge \sim p$
- (e)  $((p \wedge \sim q) \implies (r \wedge \sim r)) \implies (p \implies q)$

2. (Number 9 page 30 of Greenberg) Can you think of any way to prove from the postulates in chapter 1 that for every line  $l$

- (a) There exists a point lying on  $l$  ?
- (b) There exists a point not lying on  $l$  ?

3. (Number 12 page 31 of Greenberg) What is the flaw in the ‘proof’ that all triangles are isosceles?

**0.1 Type II Problems**

1. In each of the below, give examples of sets  $A, B$  that satisfy the specified property.

- (a)  $A \subset B$
- (b)  $A \not\subseteq B$
- (c)  $A \in B$
- (d)  $A \notin B$
- (e)  $A \subset A$
- (f)  $A \not\subseteq A$
- (g)  $A \notin A$
- (h)  $A \in A$

2. Let  $S$  be the collection of all sets that do not contain themselves as elements. Is  $S$  in  $S$  or is it not in  $S$ ?

- (a) Look up Russell’s paradox and give a brief explanation of how mathematicians now deal with this paradox in set theory.

3. Show that it is impossible for any set to be in one-to-one correspondence with its power set. (Include infinite sets in your presentation.)
4. Do Major exercise 1 page 31 of Greenberg.
5. Do Major exercise 2 page 32 of Greenberg.