Physics 122MA/Math 221PH
Spring 2000 Quiz 5 Name ____________________

Instructions: We encourage you to work with others in your assigned group on this quiz. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is important that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This quiz is due at the beginning of class on Friday, March 2.

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1. (a) Find the potential energy of a collection of total charge $Q$ spread uniformly over the surface of a sphere of radius $R$. Express your answer as a function of $Q$ and $R$. *Hint:* Compute the work done in assembling the charge in this configuration by bringing small amounts of charge $\Delta q$ onto the surface from an infinite distance away. Assume that the potential energy an infinite distance away from the final charge configuration is zero.

   (b) Repeat (a) but this time assemble the charge uniformly throughout the volume of the sphere of radius $R$. Again express your answer as a function of $Q$ (the total charge) and $R$. [This problem is of practical importance in understanding the atomic nucleus.]

2. In class, we outlined the procedure for finding the second derivative of a function on a single variable when using the chain rule. Specifically,

$$
\begin{aligned}
y &= f\left(g\left(x\right)\right) \\
\frac{dy}{dx} &= f'\left(g\left(x\right)\right)g'\left(x\right) \\
\frac{d^2y}{dx^2} &= \frac{d}{dx}\left[f'\left(g\left(x\right)\right)g'\left(x\right)\right] \\
& \frac{d}{dx}\left[f'\left(g\left(x\right)\right)\right]g'\left(x\right) + f'\left(g\left(x\right)\right)\frac{d}{dx}\left[g'\left(x\right)\right] \\
&= \left(f''\left(g\left(x\right)\right)g'\left(x\right)\right)g'\left(x\right) + f'\left(g\left(x\right)\right)g''\left(x\right) \\
&= f''\left(g\left(x\right)\right)\left(g'\left(x\right)\right)^2 + f'\left(g\left(x\right)\right)g''\left(x\right).
\end{aligned}
$$

If $z = f\left(x, y\right)$, where $x = g\left(u, v\right)$, and $y = h\left(u, v\right)$, use the above as a template to derive the formula for

$$\frac{\partial^2 z}{\partial u \partial v}.$$

Hint: The notation is simpler if you use the Leibniz forms $\left(\frac{\partial}{\partial v}\right)$ of the derivatives.