

Instructions: We encourage you to work with others in your assigned group on this quiz. You should write your solution neatly using complete sentences that incorporate all symbolic mathematical expressions into the grammatical structure. Include enough detail to allow a fellow student to reconstruct your work, but you need not show every algebraic or arithmetic step. It is important that you do your own writing, even if you have worked out the details with other people. All graphs should be done carefully on graph paper or drawn by a computer. This quiz is due at the beginning of class on Friday, March 30.

1. Derive Gauss's law for gravitation. *Hint:* Consider a point particle of mass m and find the gravitational field \vec{g} in the space around the particle. Then compute the flux of the gravitational field

$$\Phi_g = \oiint \vec{g} \cdot \hat{n} dA$$

through a Gaussian sphere, and argue that your result is true in general for a Gaussian surface enclosing a mass.

2. Consider the vector field

$$\vec{F}(x, y) = \left\langle \frac{-y}{x^2 + y^2}, \frac{x}{x^2 + y^2} \right\rangle.$$

- (a) Show $f(x, y) = -\arctan(x/y)$ satisfies

$$\frac{\partial f}{\partial x} = \frac{-y}{x^2 + y^2} \quad \text{and} \quad \frac{\partial f}{\partial y} = \frac{x}{x^2 + y^2}$$

(2 points)

- (b) Use formula 22.5 on page 742 of the text to compute $\int_{C_1} \vec{F} \cdot d\vec{s}$ where C_1 is the circle $(x - 2)^2 + (y - 3)^2 = 1$ parametrized by $\vec{r}(t) = \langle 2 + \cos(t), 3 + \sin(t) \rangle$; $t \in [0, 2\pi]$. Explain why this is exactly the result guaranteed by the fundamental theorem of line integrals. (2 points)
- (c) (4 points) Use formula 22.5 on page 742 of the text to compute $\int_C \vec{F} \cdot d\vec{s}$, where C is the circle parametrized by $\vec{r}(t) = \langle \cos(t), \sin(t) \rangle$; $t \in [0, 2\pi]$. (2 points)
- (d) Carefully explain why part (c) does or does not contradict the fundamental theorem of calculus for line integrals. (4 points)