SPRING SEMESTER 1996
EXAMINATION ONE
HONORS 213

Thompson 127, 12:00 P.M. Bryan Smith

The beginning of wisdom is the definition of terms. -**Socrates**

When taking the examination, be sure to give as complete answers as you can. You need not provide as much detail as that necessary in a homework assignment but your solutions should be complete and easy to follow.

I 5 points

Give an example of a conditional statement, its converse and its contrapositive.

II 5, 5 points 1. Which parallel property does this logical statement encode?

$$\forall l \quad \forall \mathbf{P} \quad \forall m \quad (\sim (\mathbf{P}_{\mathrm{I}} l) \wedge (\mathbf{P}_{\mathrm{I}} m) \wedge (m \neq l)) \Longrightarrow (\exists \mathbf{Q} \ni (\mathbf{Q}_{\mathrm{I}} l) \wedge (\mathbf{Q}_{\mathrm{I}} m))$$

2. What is the negation of the above logical statement.

III 15 points

Using any result up to and including Proposition 2.4, prove the following.

For every point $\mathbf{P}$ there exist at least two lines through $\mathbf{P}$.

IV 20 points

Using any previous result, prove the following portion of Pasch's Theorem.

If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are distinct noncollinear points and l is any line intersecting $\mathbf{AB}$ in a point between $\mathbf{A}$ and $\mathbf{B}$, then l also intersects either $\mathbf{AC}$ or $\mathbf{BC}$.

V 20 points

Let $\mathcal{M}$ be a finite projective plane. Prove that all lines in $\mathcal{M}$ have the same number of points lying on them.

VI 20 points

Using any result up to and including Proposition 3.2, prove the following part of Proposition 3.3.

If $\mathbf{A} * \mathbf{B} * \mathbf{C}$ and $\mathbf{A} * \mathbf{C} * \mathbf{D}$, then $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are distinct, collinear points.

VII 10 points

Use the Forward-Backward method to analyze the following problem (the converse of the Same Side Lemma)

Given that $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are collinear points, l is a line other than $\overleftrightarrow{\mathbf{AC}}$ passing through $\mathbf{C}$ and points $\mathbf{A}$ and $\mathbf{B}$ are on the same side of l. Then $\mathbf{A} * \mathbf{B} * \mathbf{C}$.