February 4, 2001

Name

SPRING SEMESTER 1994
EXAMINATION ONE
HONORS 213-B

Thompson 228, 9:00 A.M. Bryan Smith

> Do not imagine that Mathematics is hard and crabbed, and repulsive to common sense. It is merely the etherealization of common sense. -**Lord Kelvin**
>
> I think that one possible definition of our modern culture is that it is one in which nine-tenths of our intellectuals can't read any poetry. -**Randall Jarrell**

When taking the examination, be sure to give as complete answers as you can. You need not provide as much detail as that necessary in a homework assignment but your solutions should be complete, easy to follow, and contain justifications for each step. Since this is an in-class examination. you need not worry about following the rules outlined in the "Tips on Writing Mathematics" handout.

I 5, 5, 5 points

1. Give an example of an implication but write it in English and not in mathematical or logical notation.

2. State the converse to the above implication (again in English).

3. State the contrapositive to the first implication (in English).

II 5, 10, 5 points Find the negation of the following expressions:

1. $\forall l \forall m \exists P[(P \in l) \vee (P \in m)]$

2. $\forall l \forall P \exists m \Big\{ [(P \notin l) \wedge (P \in m) \wedge (l \parallel m)] \wedge \forall n \Big[((P \in n) \wedge (l \parallel n)) \Rightarrow (n = m) \Big] \Big\}$

3. Every line has at least two points on it.

III 10 points Is the following a tautology? Justify your answer.

$[p \wedge (p \Rightarrow q)] \Rightarrow q$.

IV **10 points** State Playfair's version of Euclid's fifth postulate.

V **15 points** In this problem you get to use the Theorem: "iF $a \neq 0$, tehn every equation of the form $ax + b = 0$ has a solution." Prove that every equation of this form has at most one solution. [Hint: Try an indirect proof and remember that you do not need to use symbols.]

VI **10 points** Rephrase the following sentence in the form $\forall x(p_x \Rightarrow q_x)$ and determine its truth value if the universe is the set of all phone calls.

"Every phone call I make during this examination will be to Hillary Clinton."

VII 10 points Briefly explain the process involved in a proof by contradiction. Discuss your own thoughts as to whether or not this process seems valid as a logical argument. [Don't get carried away and write too much.]

VII 10 points Give two examples specifying either how the axiomatic method and deductive logic would be useful in your other studies or how they would be inappropriate in those studies.