Mathematics 434 Spring 2001 Problem Sheet 2

February 5, 2001

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Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"*Do not imagine that Mathematics is hard and crabbed, and repulsive to common sense. It is merely the etherealization of common sense.*" – Lord Kelvin

Problems

1. Prove **directly** that the bilinear form represented by the matrix $\begin{bmatrix} a & b \\ b & d \end{bmatrix}$ is positive definite if and only if $a > 0$ and $ad - b^2 > 0$.

2. Let $\langle , \rangle$ be a symmetric bilinear form on a vector space V over a field F. The function $q : V \to F$ defined by $q(v) = \langle v, v \rangle$ is called the **quadratic form** associated with the bilinear form. Show how to recover the bilinear form from q, if the field is R or C, by expanding $q(v + w)$.

3. The vector space P_n of all polynomials of degree less than or equal to n has a bilinear form defined by
$$\langle f, g \rangle = \int_{-1}^{1} f(x) g(x)\, dx.$$
Find an orthonormal basis for
   (a) P_1
   (b) P_2
   (c) P_3
   (d) Compute the projection (using this bilinear form) of $h(x) = e^x$ onto P_3 and compare the accuracy of that approximation to h with the $3'$rd and 6' th degree Taylor approximations to h (centered at 0). Use careful graphs to justify your comparison. (Or if you prefer a more analytic error analysis, compute $\int_{-1}^{1} |h(x) - q(x)|\, dx$ where q is your approximation.

4. Let $V = R^{2\times 2}$ be the vector space of all real 2×2 matrices.
   (a) Determine the matrix of the bilinear form $\langle A, B \rangle = trace(AB)$ on V with respect to the standard basis $\{e_{ij}\}$.
   (b) Determine the signature of this form.
   (c) Find an orthogonal basis for this form.
   (d) Determine the signature of this form on the subspace of V of matrices with trace 0.