Mathematics 434   Spring 2001   Problem Sheet 7

March 9, 2001

<div style="text-align:right">_____
Name</div>

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"all ignorance toboggans into know and trudges up to ignorance again." – e.e.cummings, 1959

1 Problems

1. Do **one** of the following.

   (a) Determine the structure of the ring $\mathbf{Z}\left[x\right]/\left(x^2+3,p\right)$ where

      i. $p=3$

      ii. $p=5$

   (b) Describe the ring $\mathbf{Z}\left[i\right]/\left(2+i\right).$

2. Describe the ring obtained from $\mathbf{Z}$ by adjoining an element α satisfying the two relations $2\alpha-6=0$ and $\alpha-10=0.$

3. Suppose we adjoin an element α to $\mathbf{R}$ satisfying the relation $\alpha^2=1.$ Prove the resulting ring is isomorphic to the product ring $\mathbf{R}\times\mathbf{R}$, and find the element of $\mathbf{R}\times\mathbf{R}$ which corresponds to $\alpha.$

4. Let α denote the residue of x in the ring $R'=\mathbf{Z}\left[x\right]/\left(x^4+x^3+x^2+x+1\right).$ Compute the expressions for $\left(\alpha^3+\alpha^2+\alpha\right)\left(\alpha+1\right)$ and α^5 in terms of the basis $\left(1,\alpha,\alpha^2,\alpha^3,\alpha^4\right).$

5. Do **one** of the following.

   (a) In each case describe the ring obtained from $\mathbf{Z}$ by adjoining an element α satisfying the given relation.

      i. $\alpha^2+\alpha+1=0$

      ii. $\alpha^2+1=0$

   (b) Let $R=\mathbf{Z}/\left(10\right).$ Determine the structure of the ring R' obtained from $\mathbf{Z}$ by adjoining element α satisfying each relation.

      i. $2\alpha-6=0$

      ii. $2\alpha-5=0.$

6. Describe the ring obtained from $\mathbf{Z}/12\mathbf{Z}$ by adjoining an inverse of 2.