

November 10, 2000

---

Name

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**

*"Iron rusts from disuse; stagnant water loses its purity and in cold weather becomes frozen; even so does inaction sap the vigor of the mind."* – Leonardo da Vinci

**Problems**

1. Do **one** of the following.
  - (a) Let  $G = D_4$  be the dihedral group of symmetries of the square.
    - i. What is the stabilizer of a vertex? Of an edge?
    - ii.  $G$  acts on the set of two elements consisting of the diagonal lines. What is the stabilizer of a diagonal?
  - (b) Let  $G = GL(n, R)$  operate on the set  $S = R^n$  by left multiplication.
    - i. Describe the decomposition of  $S$  into orbits for this operation.
    - ii. What is the stabilizer of  $e_1$ ?
2. Do **one** of the following.
  - (a) Let  $G$  be a group and let  $H$  be the cyclic subgroup generated by an element  $x$  of  $G$ . Show that if left multiplication by  $x$  fixes every coset of  $H$  in  $G$ , then  $H$  is a normal subgroup of  $G$ .
  - (b) A map  $\phi : S \rightarrow S'$  of  $G$ -sets is called a **homomorphism** of  $G$ -sets if  $\phi(gs) = g\phi(s)$  for all  $s \in S$  and all  $g \in G$ . Let  $\phi$  be such a homomorphism. Prove the following.
    - i. The stabilizer  $G_{\phi(s)}$  contains the stabilizer  $G_s$ .
    - ii. The orbit of an element  $s \in S$  maps onto the orbit of  $\phi(s)$ .