# 1 Mathematics 433 Fall 2001 Problems to Turn in: 2

**September 7, 2001**

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**Name**

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
*"The one real object of education is to have a man in the condition of continually asking questions."*
-Bishop Mandell Creighton

## Problems

1. Do both of the following:

   (a) Prove that if $G$ is a group with the property that the square of every element is the identity, then $G$ is abelian.

   (b) Let $G$ be a finite group. Show that the number of elements $x$ of $G$ such that $x^3 = e$ is odd. Show that the number of elements $x$ of $G$ for which $x^2 \neq e$ is even.

2. Do any two of the following

   (a) Prove that every subgroup of a cyclic group is cyclic.

   (b) Prove that the set of elements of finite order in an abelian group is a subgroup.

   (c) If $H$ and $K$ are subgroups of a group $G$, show that $H \cap K$ is a subgroup of $G$. Adapt your proof to show that the intersection of any number of subgroups of $G$, finite or infinite, is again a subgroup of $G$. Notational hint: Let $C$ be a collection of subgroups of $G$. Then we can denote the intersection of all the subgroups in $C$ by

   $$\bigcap_{H \in C} H$$

   .

3. Show by example that the product of elements of finite order in a nonabelian group need not have finite order.