# 1 Mathematics 433 Fall 2001 Assignment 1

**August 22, 2001**

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**Name**

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use the results of discussion, a text, notes, or technology. **Only write on one side of each page.**
"*Simple solutions seldom are. It takes a very unusual mind to undertake analysis of the obvious.*" — Alfred North Whitehead

## Problems

**When doing problems associated with matrices, you are not restricted to the material covered, so far, in our review.**

1. You must do this problem. Do **two** of the following.

   (a) (Vandermonde Determinant)

   i. Prove that $\det \begin{bmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix} = (b-a)(c-a)(c-b)$.

   ii. $(*)$ Prove an analogous formula for $n \times n$ matrices by using induction and row operations (in a clever fashion) to clear out the first column.

   (b) Find a formula for $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}^n$, and prove it by induction.

   (c) Use induction to compute the determinant of $A = \begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & -1 & & \\ & & & \ddots & & \\ & & & -1 & 2 & -1 \\ & & & & -1 & 2 \end{bmatrix}$.

   (d) Use Mathematical Induction to prove that if $A_1, \cdots, A_n$ are square, invertible, $m \times m$ matrices then the product $A_1 \cdots A_m$ is also invertible and

   $$(A_1 \cdots A_m)^{-1} = A_m^{-1} \cdots A_1^{-1}.$$

2. Do one of the following.

   (a) Prove that the Second Principle of Mathematical Induction implies the First Principle of Mathematical Induction.

   (b) $(*)$ Let $A, B$ be $m \times n$ and $n \times m$ matrices. Prove $I_m - AB$ is invertible if and only if $I_n - BA$ is invertible.

3. Do **both** of the following.

   (a) Let $a, b$ be elements of a group $G$. Show that the equation $ax = b$ has a unique solution in $G$.

   (b) Let $G$ be a group, with multiplicative notation. Define an **opposite group** $G^0$ with law of composition $a \circ b$ as follows: The underlying set is the same as for $G$, but the law of composition is the opposite; that is, define $a \circ b = ba$. Prove that this defines a group.

4. Show that for any set (including infinite sets) $A$ it is not the case that $A$ is in one-one correspondence with the power set of $A$, $P(A)$.