

1 Additional Exercises: Restrictions of Homomorphisms

- Let G and G' be finite groups whose orders have no common factor. Prove the only homomorphism from G to G' is the trivial one

$$\phi(x) = e' \quad \text{for all } x \in G.$$

- Let H and K be subgroups of a group G .
 1. Prove the intersection $xH \cap yK$ of two cosets of H and K is either empty or a coset of the subgroup $H \cap K$.
 2. Prove if H, K have finite index in G then $H \cap K$ also has finite index.
- Let $\phi : G \rightarrow G'$ be a group homomorphism with kernel K . Let H be another subgroup of G . Describe $\phi^{-1}(\phi(H))$ in terms of H and K .
- Prove that a group of order 30 can have at most 7 subgroups of order 5.