1 Additional Exercises: Isomorphisms

- Let a, b be elements of a group G, and let $a' = bab^{-1}$. Prove $a = a'$ if and only if a and b commute.

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  1. Let $b' = aba^{-1}$.Prove that $(b')^n = ab^n b^{-1}$
  2. Prove if $aba^{-1} = b^2$,then $a^3ba^{-3} = b^8$.

- Let $\phi : G \to G'$ be an isomorphism of groups, let $x, y \in G$ and $x' = \phi(x)$ and $y' = \phi(y)$.

  1. Prove the orders of x and x' are equal.
  2. Prove if $xyx = yxy$,then $x'y'x' = y'x'y'$.
  3. Prove $\phi(x^{-1}) = (x')^{-1}$.

- Prove $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ are conjugate elements in $GL(2, R)$ but are not conjugate in $SL(2, R)$.

- Prove the map $\phi : GL(n, R) \to GL(n, R)$ given by $\phi(A) = (A^t)^{-1}$ is an automorphism.