1 Additional Exercises

1.1 Counting Formula

1. Compute the order of the group of symmetries of a dodecahedron, when orientation reversing symmetries such as reflections in planes, as well as rotations are allowed. Do the same for the symmetries of a cube.

2. Let G be the group of rotational symmetries of a cube. Let S_e, S_f, S_v be the sets of edges, faces and vertices of the cube, respectively. Let H_e, H_f, H_v be the stabilizers of a particular edge, face and vertex, respectively. Determine the formulas that represent the decomposition of each of the three sets S_e, S_f, S_v into orbits for each of the three subgroups.

1.2 Operations of a group on itself

1. GIven a group G, does the mapping $f : G \times G \to G$ given by $f(g, x) = xg^{-1}$ define a group action of G onto itself?

2. Determine the class equation for each of the following groups.

   (a) The quaternion group.

   (b) The Klein four group.

   (c) The dihedral group D_5.

   (d) The dihedral group D_6

   (e) The dihedral group D_n

1.3 Class Equation of Icosahedral Group

1. Identify the intersection $I \cap O$ when the dodecahedron and cube are as in Figure 2.7 which was passed out in class. Here, I is the group of 60 rotational symmetries of the dodecahedron and O is the group of 24 rotational symmetries of the cube.

2. Two tetrahedra can be inscribed into a cube C, each one using half of the vertices. Relate this to the inclusion $A_4 \subset S_4$. Here S_4 is the symmetric group of all permutations of the elements $\{1, 2, 3, 4\}$ and A_4 is the normal subgroup of S_4 consisting of the even permutations in S_4. Recall that a permutation p is even if its matrix P has the property that $\det(P) = 1$.

3. Prove or disprove: An abelian group is simple if and only if it has prime order.