1 Additional Exercises: Group Actions and Cosets

1. Determine the group of automorphisms of the following groups.

   (a) C_4

   (b) C_6

   (c) $C_2 \times C_2$

2. Let S be a set on which the group G operates. Prove the relation $s \sim s'$ if $s' = gs$ for some $g \in G$ is an equivalence relation.

3. Let $\phi : G \to G'$ be a homomorphism, and let S be a set on which G' acts. Show how to define a group action of G on S using the homomorphism ϕ.

4. Do both of the following

   (a) Describe the orbit and the stablilzer of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ under conjugation in $GL(2, R)$.

   (b) FInd the number or elements in the orbit of the matrix $\begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ under conjugation in $GL(2, F_3)$. Here F_3 is the set of integers $\{0, 1, 2\}$ with both addition and multiplication done modulo 3. Thus, in F_3, we have $2^3 = 8 = 2(3) + 2 = 2 \pmod 3$ and $1 + 2 = 0 \pmod 3$.

5. What is the stabilizer of the coset aH for the action of G on G/H?

6. Describe all ways in which S_3 can operate on a set of four elements.