1 Additional Exercises: Equivalence Relations

- Prove the nonempty fibres of a map ϕ form a partition of the domain.
- Let S be a set of groups. Prove the relation $G\tilde{}H$ if G is isomorphic to H is an equivalence relation on S.
- Let H be a subgroup of a group G.Prove the relation defined by the rule $a\tilde{}b$ if $b^{-1}a \in H$ is an equivalence relation on G.
- With each of the following subsets R of the (x, y) - plane, determine which of the three defining axioms of an equivalence relation where $x\tilde{}y$ if and only if $(x, y) \in R$.
  1. $R = \{(s, s) : s \text{ a real number}\}$
  2. $R = \{\}$
  3. $R = \{(x, y) : y = 0\}$
  4. $R = \{(x, y) : xy + 1 = 0\}$
  5. $R = \{(x, y) : x^2y - xy^2 - x + y = 0\}$
  6. $R = \{(x, y) : x^2 - xy + 2x - 2y = 0\}$.
- Draw the fibres of the map from the (x, z) - plane to the y - axis defined by the map $y = zx$.