Mathematics 300 **Spring 2005** **Exam 2**

Mar 8, 2005

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Name

Directions: Only write on one side of each page.

I. Do any five (5) of the following

1. Prove the "Opposite Side Lemma": Given $A * B * C$ and l any line other than $\overleftrightarrow{AB}$ meeting line $\overleftrightarrow{AB}$ at point B. Then points A and C are on opposite sides of line l.

2. Let M be a projective plane and let M' be the interpretation of the point, line, and incident where the points of M' are interpreted to be the lines of M, the lines of M' are interpreted to be the points of M, and a point and line of M' are incident if and only if the corresponding line and point of M are incident.

   Do **one** (1) of the following.

   (a) Prove that Incident Axiom 3 "makes sense" in M'.

   (b) Prove that each line of M' is incident with at least three points of M'.

3. Using any previous results, prove Proposition 3.18.

   If in $\triangle ABC$ we have $\measuredangle B \cong \measuredangle C$, then $AB \cong AC$ and $\triangle ABC$ is isosceles.

4. Justify each step in the following proof of Proposition 3.11.

   (a) Assume on the contrary that BC is not congruent to EF.

   (b) Then there is a point G on ray $\overrightarrow{EF}$ such that $BC \cong EG$.

   (c) $G \neq F$.

   (d) Since $AB \cong DE$, adding gives $AC \cong DG$.

   (e) However, $AC \cong DF$.

   (f) Hence $DF \cong DG$.

   (g) Therefore, $F = G$.

   (h) Our assumption has led to a contradiction.

   (i) Hence, $BC \cong EF$.

5. Using any results up to and including Proposition 3.7, prove the first part of Proposition 3.8.

   If D is in the interior of $\measuredangle CAB$ then so is every other point of ray $\overrightarrow{AD}$ except A.

6. Do **one** (1) of the following that was not a homework problem assigned to you.

   (a) Using any results up to and including Proposition 3.9 prove the following.
   No line can be contained in a triangle.

   (b) Using any results (on the handout sheet of propositions) up to and including Proposition 3.13 (3), prove Proposition 3.13 (4).
   If $AB < CD$ and $CD < EF$, then $AB < EF$. [Note: Propositions 3.13 (c), and 3.13 (d) in the textbook are numbered 3.13 (3) and 3.13 (4) in the handout sheet.]