Mathematics 300 **Spring 2003** **Exam 1**

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

Problems

1. (10 points) Negate the following logical statement to the point there is no longer an implication.

$$\forall\varepsilon\ \exists N \quad (n > N) \Longrightarrow (|a_n - L| < \varepsilon)\,.$$

2. (10 points) Prove the following logical statement is a tautology.

$$(\sim q \wedge (p \Longrightarrow q)) \Longrightarrow \sim p$$

3. (20 points) Using any previous result and the Axioms, formally prove Proposition 2.4. For every point there is at least one line not passing through it. Be sure to justify every step.

4. (15, 5 points) Here is an interpretation of the undefined terms of incidence geometry: Fix a circle in the Euclidean plane. Every Euclidean point that is **inside** the circle is interpreted to be a "'point". Every Euclidean chord of the circle is interpreted to be a "line". "Incidence" means that a "point" lies on a "line" in the usual Euclidean sense. [**Definition:** The portion of a Euclidean line that lies inside a circle is called **chord** of that circle.]

   (a) Which of the axioms of Incidence geometry are satisfied by this interpretation? Explain.

   (b) Does this interpretation have a parallel property? If so, is it the elliptic, Euclidean, or hyperbolic parallel property? Explain.

5. (20 points) Recall that a projective plane is a model of incidence geometry satisfying the elliptic parallel property and in which every line has at least three points incident with it.

   Let M be a projective plane and let $M\,'$ be the interpretation of the undefined terms obtained by interpreting the points of $M\,'$ to be the lines of M,the lines of $M\,'$ points of M, and $M\,'$ points and lines to be incident exactly when the corresponding lines and points of M are incident.

   (a) Using properties known to be true about M, prove that $M\,'$ satisfies Incidence Axiom 2.

   (b) Using properties known to be true about M, prove that $M\,'$ satisfies Incidence Axiom 1.

   (c) Using properties known to be true about M, prove that $M\,'$ satisfies the elliptic parallel property.

6. Do **one** of the following.

   (a) (20 points) Modelling problem:

      i. Explain how one uses models of an axiomatic system to prove a given statement is **independent** of that axiomatic system.

      ii. Use two models of incidence geometry to show that the following statement is independent of incidence geometry.
      Given distinct lines l, m, and n.If l is parallel to m and m is parallel to n, then l is parallel to n.

   (b) (20 points) What is the smallest number of lines possible in a model of incidence geometry in which there are exactly 5 points? Include a careful argument supporting your claim (but you need not provide a formal proof.)