Smith **Math 290-A** **Fall 2008**

Final Exam **December** 17, 2008

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Define all three of the following.

D.1. [6 points] The coordinate transformation ρ_B where $B = \left\{\vec{b}_1, \cdots, \vec{b}_n\right\}$ is a basis for the vector space V.

D.2. [7 points] A linearly dependent subset S of a vector space V.

D.3. [7 points] The geometric multiplicity of an eigenvalue.

Do one (1) of these "Computational" problems

C.1. [15 points] Given a vector space W and two subsets of W: $S = \{\vec{w}_1, \vec{w}_2, \vec{w}_3, \vec{w}_4\}$ and $T = \{\vec{w}_1 + 2\vec{w}_2 + 3\vec{w}_3 + 4\vec{w}_4,$ If S is linearly independent in W either prove that T is also linearly independent or write one of the four vectors in T as equal to a linear combination of the other three vectors in T. Show all work.

C.2. [15 points] Given the linear transformation $T : M_{22} \to M_{22}$ given by $T(A) = A + A^t$. Find the matrix representation $M^T_{B,C}$ where B is the standard basis of $M_{22}$and $C = \left\{ \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \right\}$

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 Points] A certain 5×5 matrix C can be written as $C = AB$ where A is 5×4 and B is 4×5. Explain how you know that $\det(C) = 0$.

M.2. [15 Points] Prove Theorem AIU from our textbook.

Theorem AIU: Suppose that V is a vector space. For each $\vec{u} \in V$, the additive inverse, $-\vec{u}$ is unique. (You may **not** use the fact that $-\vec{u} = (-1)\,\vec{u}$.)

M.3. [15 Points] Prove Theorem CILTI from the textbook.

Theorem CILTI: Suppose that $T : U \to V$ and $S : V \to W$ are both injective linear transformations. Then $(S \circ T) : U \to W$ is an injective linear transformation. You may use, without proving it, the fact that $S \circ T$ is a linear transformation.

Do three (3) of these "Other" problems

T.1. [15 Points] Recall the definition given in class that a square matrix A of size n is **skew-symmetric** if $A^t = -A$. Prove that if n is an odd integer then A is not invertible. [Hint: Consider determinants.]

T.2. [15 Points] Suppose $T : U \to V$ is a function that satisfies the single condition$T(\alpha\vec{x} + \vec{y}) = \alpha T(\vec{x}) + T(\vec{y})$ for every $\vec{x}, \vec{y}$ in U and every α in $\mathbf{C}$.Prove that $T\left(\vec{0}\right) = \vec{0}$. You may **not** use the fact that T is a linear transformation.

T.3. [15 Points] Prove that the matrices $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are similar by finding a matrix S for which $A = S^{-1}BS$.

T.4. [15 points] Let $B = \left\{e^x, xe^x, x^2e^x\right\}$ be a basis for the subspace V of the vector space F of functions with domain and codomain the set of complex numbers: $F = \{f \mid f : \mathbf{C} \to \mathbf{C}\}$.

1. Find the matrix representation $M^T_{B,B}$ of the linear transformation $T : V \to V$ defined by $T(f) = f\,'$.
2. Use this matrix representation to find the kernel of T, $\ker(T)$.

You must do both of these problems ON THIS SHEET

R.1. [15 points] Prove that the set $V = \left\{ \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbf{C}^3 : \ 2x_1 - 7x_2 + x_3 = 0 \right\}$ is a subspace of $\mathbf{C}^3$ by applying the three-part test of Theorem TSS. Write your proof according to the standards of this semester's writing exercises.

R.2. [15 points] Part of Theorem NPNT ("Nonsingular Products, Nonsingular Terms") says: If A and B are square matrices of the same size, and AB is nonsingular, then B is nonsingular. Construct a proof by contradiction of this fact and write your proof according to the standards of this semester's writing exercises. (You may **not** do this problem by simply quoting Theorem NPNT.)