Smith Math 290 Fall 2011

Exam 4

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do and two (2) of these "Computational" problems

C.1. [20 points] Let $A = \begin{bmatrix} 0 & 2 & -2 \\ 2 & 0 & 3 \\ 0 & 0 & 3 \end{bmatrix}$.If possible, find a matrix S for which $S^{-1}AS$ is diagonal.

C.2. [20 points] Compute the matrix representation $M^T_{B,B}$ of the linear transformation $T : M_{22} \to M_{22}$ defined by $T(A) = A + A^t$. Use $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and the basis $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$

C.3. [10, 10 points] Suppose the function $T : \mathbf{C}^3 \longrightarrow P_2$ is defined by $T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix}.\right) = (a+b) + (b+2c)\,x + (-a+2c)\,x^2$

1. Show that T is a linear transformation.
2. Find a basis for the range of T.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Prove Theorem ILTLT: Suppose that $T : U \longrightarrow V$ is an invertible linear transformation. then the function $T^{-1} : V \longrightarrow U$ is a linear transformation.

M.2. [8, 7 points] Prove **two** (2) of the following

1. If A is similar to B and A is invertible, then B is invertible.
2. If A is diagonalizable then A^2 is diagonalizable.
3. If B is nonsingular show that AB is similar to BA

M.3. [15 points] Prove Theorem KILT: Suppose that $T : U \longrightarrow V$ is a linear transformation. Then T is injective **if and only if** the kernel of T is trivial, $K(T) = \left\{\vec{0}\right\}$.

Do any two (2) of these "Other" problems

T.1. [15 points] Prove that if W is a subspace of U and $T : U \to V$ is a linear transformation, then the set $T(W) = \{T(\vec{u}) \mid \vec{u} \in W\}$ is a subspace of V.

T.2. [15 points] Suppose U, V, and W are vector spaces and $T : U \longrightarrow V$, $S : V \longrightarrow W$ are linear transformations. Prove $\ker(T) \subseteq \ker(S \circ T)$.

T.3. [15 points] Suppose U and V are vector spaces where $\dim(U) = n$, $\dim(V) = m$ and $n > m$. Prove there cannot be linear transformations $T : U \to V$ and $S : V \to U$ where $S \circ T = I_U$. Here $I_U : U \to U$ is the identity map.

T.4. [15 points] Prove that if $T : V \to V$ and $Z : V \to V$ are linear transformations where $T \circ T \circ T \circ T = Z$ and $Z(\vec{v}) = \vec{0}$ for every $\vec{v} \in V$, then T is not invertible.