Smith **Math 290** **Fall 2010**

Exam 4

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any two (2) of these "Computational" problems

C.1. [15 points] The sets $B = \left\{1+x,\ \ 2+x^2,\ \ 3+x+x^2\right\}$ and $D = \left\{3,\ \ 2-x,\ \ 1-x^2\right\}$ are both bases for P_2.Compute the change of basis matrix $C_{B,D}$ and use it to compute $\rho_D\left(5\left(1+x\right)+4\left(2+x^2\right)\right)$

C.2. [15 points] Consider a linear transformation $T\left(\overrightarrow{x}\right) = A\overrightarrow{x}$ from $\mathbf{C}^n$ to $\mathbf{C}^n$. The set $B = \{-\overrightarrow{e_1}, \overrightarrow{e_2}, -\overrightarrow{e_3}, \cdots, (-1$
a basis of $\mathbf{C}^n$. Describe the entries of $M^T_{B,B}$ in terms of the entries of A.

C.3. [15 points] Find the matrix representation $M^T_{B,B}$ of the linear transformation $T : P_2 \to P_2$ defined by $T\left(f\left(x\right)\right) = f\left(x-2\right)$ where $B = \left\{1, x, x^2\right\}$ is the standard basis of P_2.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Prove Theorem ILTLT: Suppose that $T : U \to V$ is an invertible linear transformation. Then $T^{-1} : V \to U$ is a linear transformation.

M.2. [15 points] Prove either half of Theorem ILTB: Suppose that $T : U \to V$ is a linear transformation and $B = \{\vec{u}_1, \vec{u}_2, \cdots, \vec{u}_n\}$ is a basis for U. Then T is injective if and only if the set $C = \{T\left(\vec{u}_1\right), T\left(\vec{u}_2\right), \cdots, T\left(\vec{u}_n\right)\}$ is linearly independent.

M.3. [15 points] Prove that if A is diagonalizable, then A^{-1} is diagonalizable.

M.4. [15 points] Prove that if U and V are vector spaces, W is a subspace of V and $T : U \to V$ is a linear transformation, then $T^{-1}\left(W\right)$ is a subspace of U. (The preimage of W is a subspace of U.)

Do any two (2) of these "Other" problems

T.1. [15 points] If $T : V \to V$ is a linear transformation, prove that $\left(T \circ T\right)\left(\vec{v}\right) = \vec{0}$ for every vector $\vec{v} \in V$ if and only if $R\left(T\right) \subseteq K\left(T\right)$ (the range of T is a subset of the kernel of T)

T.2. [15 points] Let $B = \left\{1, x, x^2\right\}$ be the standard basis of P_2, $f\left(x\right) = 2+x$, and $V = \{g \in P_2 : \langle\rho_B\left(g\right), \rho_B\left(f\right)\rangle = 0$
Find a basis for the vector space V (you do not have to show that V is a subspace of P_2 – just find a basis.)

T.3. $[5, 10$ points$]$ Let V be a vector space and $Z = \left\{\vec{0}_V\right\}$. Define a function $I_Z : Z \to V$ by $I_Z\left(\vec{0}_V\right) = \vec{0}_V$.

1. Prove that I_Z is a linear transformation. (You need not prove that Z is a vector space.)
2. Prove that $T : V \to V$ is injective if and only if $K(T) \subseteq R(I_Z)$ (the kernel of T is a subset of the range of I_Z.)

$$\vec{0} \xrightarrow{I_Z} V \xrightarrow{T} V$$