Smith **Math 290** **Fall 2009**

Exam 4

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any two (2) of these "Computational" problems

C.1. [15 points] Verify that $T : P_2 \to P_2$ defined by $T(p(x)) = (x+2)\,p'(x)$ is a linear transformation.

C.2. [15 points] Compute the matrix representation $M_{B,B}^{T}$ of the linear transformation $T : M_{22} \to M_{22}$ defined by $T(A) = A + A^t$. Use $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and the basis $B = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$

C.3. [15 points] Define a linear transformation $T : \mathbf{C}^2 \to \mathbf{C}^2$ by $T\left(a \begin{bmatrix} 2 \\ 1 \end{bmatrix} + b \begin{bmatrix} 1 \\ -1 \end{bmatrix} \right) = a \begin{bmatrix} 6 \\ 3 \end{bmatrix} + b \begin{bmatrix} -3 \\ 3 \end{bmatrix}$. Find a matrix A with the property that $T(\vec{x}) = A\vec{x}$ for every vector $\vec{x} \in \mathbf{C}^2$.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [10, 10 points] Suppose $T : U \to V$ and $S : V \to W$ are linear transformations.

1. Prove the range of $S \circ T$ is a subset of the range of S. That is, prove $R(S \circ T) \subseteq R(S)$.
2. Prove the kernel of T is a subset of the kernel of $(S \circ T)$. That is, prove $\ker(T) \subseteq \ker(S \circ T)$.

M.2. [20 points] Suppose $T : U \to V$ and $S : U \to V$ are linear transformations and recall that the function $T + S : U \to V$ is defined by $(T+S)(\vec{u}) = T(\vec{u}) + S(\vec{u})$ for all $\vec{u} \in U$. Prove that $T + S$ is a linear transformation.

M.3. [20 points] Prove the theorem: if $T : U \to V$ is an injective linear transformation and $\dim(U) = \dim(V) = n$ then T is an isomorphism.

Do any two (2) of these "Other" problems

T.1. [15 points] Prove that if $T : V \to V$ and $Z : V \to V$ are linear transformations where $T \circ T \circ T \circ T = Z$ and $Z(\vec{v}) = \vec{0}$ for every $\vec{v} \in V$, then T is not invertible.

T.2. [15 points] Suppose $T : U \to V$ and $S : V \to U$ are linear transformations and $\dim(U) > \dim(V)$.Prove $S \circ T$ cannot be the identity map $I_U : U \to U$.

T.3. [15 points] Prove that if W is a subspace of U and $T : U \to V$ is a linear transformation, then $T(W) = \{T(\vec{w}) : \vec{w} \in W\}$ is a subspace of V.

T.4. [15 points] Prove that if X is a subspace of V and $T : U \to V$ is a linear transformation, then $T^{-1}(X) = \{\vec{u} \in U : T(\vec{u}) \in X\}$ is a subspace of U.