Smith Math 290 Spring 2013

Exam 3

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do both of these "Computational" problems

C.1. [15 points] If V is a subspace of $\mathbf{C}^n$ then $V^\perp$ is defined to be the set $V^\perp = \left\{ \vec{x} \in \mathbf{C}^n \mid \forall \vec{v} \in V \quad \langle \vec{x}, \vec{v} \rangle = \vec{0} \right\}$ .Tha is, $V^\perp$ is the set of all vectors in $\mathbf{C}^n$ that are orthogonal to every vector in V.

1. Show that $V^\perp$ is a subspace of $\mathbf{C}^n$.

C.2. [15 points] Express $4-t-t^2$ as a linear combination of the vectors in $S = \left\{1+t^2, t+t^2, 1+2t+t^2\right\}$.

Do one (1) of these "In Class, Text, or Homework" problems

1. [15 points] Show that $C(AB) \subseteq C(A)$. Here, $C(A)$ is the column space of matrix A.
2. [15 points] Prove that if matrix A is diagonalizable then A^3 is diagonalizable.

Do any two (2) of these "Other" problems

1. [20 Points] Prove that if A, B are matrices for which the product AB is defined, then $\eta(B) \leq \eta(AB)$. Here $\eta(A)$ is the nullity of A.
2. [20 Points] Let A be an $n \times n$ matrix and let λ be a nonzero eigenvalue of A. Show that if $\vec{x}$ is an eigenvector corresponding to λ then $\vec{x}$ is in the column space of A.
3. [20 Points] Prove the following by contradiction. If λ and ρ are two distinct (not equal) eigenvalues of the square matrix A, then the intersection of the eigenspaces for these two eigenvalues is trivial. That is, $E_A(\lambda) \cap E_A(\rho) = \left\{\vec{0}\right\}$.

Definitions

1. [15 points] Given a set V and an addition and scalar multiplication for elements in V, there are 10 properties that must hold for V to be a vector space. List those properties. Give the actual mathematical statements of the properties rather than the names of the properties. For example: write $\alpha(\vec{x}+\vec{y}) = \alpha\vec{x} + \alpha\vec{y}$ instead of saying "scalar multiplication distributes over vector addition".

Useful information

1. $\vec{x} \in N(A)$ iff $A\vec{x} = \vec{0}$
2. $\vec{y} \in C(A)$ iff there exists an $\vec{x}$ with $A\vec{x} = \vec{y}$