Smith **Math 290** **Fall 2011**

Exam 3

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do and two (2) of these "Computational" problems

C.1. [20 points] Prove that the set $V = \left\{A \in M_{44} : A^3 - 2A^2 + 3I_4 = O_4\right\}$ is a subspace of M_{44}.

C.2. [20 points] Find a basis for the subspace V of P_3 given by $V = \{p \in P_3 : p(1) = 0 \text{ and } p(-1) = 0\}$

C.3. [20 points] Given the matrix $A = \begin{bmatrix} 25 & -8 & 30 \\ 24 & -7 & 30 \\ -12 & 4 & -14 \end{bmatrix}$, and the fact that $P_A(x) = -\left(x^3 - 4x^2 + 5x - 2\right)$.

Compute the eigenspace $E_A(\lambda)$ where λ is the smallest eigenvalue of A.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Let U be a vector space and V, W subspaces of U of dimension 2 and 3 respectively. Let $B = \{\vec{v}_1, \vec{v}_2\}$ be a basis of V and $C = \{\vec{w}_1, \vec{w}_2, \vec{w}_3\}$ a basis for W.If the only vector common to both V and W is $\vec{0}$ prove that the set $\{\vec{v}_1, \vec{v}_2, \vec{w}_1, \vec{w}_2, \vec{w}_3\}$ is linearly independent.

M.2. [15 points] Prove Theorem ETM**:** Suppose A is a square matrix and λ is an eigenvalue of A. Then λ is an eigenvalue of A^t.

M.3. [15 points] Prove Theorem SMZE: Suppose A is a square matrix. Then A is singular if and only if $\lambda = 0$ is an eigenvalue of A. [You may **NOT** use Beezer's theorem that a matrix is non-singular if and only if 0 is not an eigenvalue of A.]

Do any two (2) of these "Other" problems

T.1. [15 points] A square matrix A is **idempotent** if $A^2 = A$. Show that the numbers 0 and 1 are the only possible eigenvalues of an idempotent matrix A.

T.2. [15 points] Prove that if U and W are both subspaces of a vector space V then the intersection, $U \cap W = \{\vec{x} \mid \vec{x} \in U \text{ and } \vec{x} \in W\}$, is also a subspace of V.

T.3. [15 points] Let A and B be $n \times n$ matrices. Show that if $\lambda = 0$ is an eigenvalue of AB, then it is also an eigenvalue of BA.

T.4. [15 points] Let A be and $n \times n$ matrix and let λ be a **nonzero** eigenvalue of A. Show that if $\vec{x}$ is an eigenvector corresponding to λ then $\vec{x}$ is in the column space of A