Smith **Math 290** **Spring 2013**

Exam 2

I affirm this work abides by the university's Academic Honesty Policy.

____

Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any two (2) of these "Computational" problems

C.1. [15 points] The vectors $\vec{u}_1, \vec{u}_2$, and $\vec{u}_3$ below form an **orthonormal** set. Use the Gram-Schmidt procedure to find a vector $\vec{u}_4$ so that $\{\vec{u}_1, \vec{u}_2, \vec{u}_3, \vec{u}_4\}$ is also an **orthonormal** set.

$$\vec{u}_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \quad \vec{u}_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}, \quad \vec{u}_3 = \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}, \vec{v}_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

The Gram-Schmidt formula for $i \geq 2$ is

$$\vec{u}_i = \vec{v}_i - \left(\frac{< \vec{v}_i, \vec{u}_1 >}{< \vec{u}_1, \vec{u}_1 >} \right) \vec{u}_1 - \cdots - \left(\frac{< \vec{v}_i, \vec{u}_{i-1} >}{< \vec{u}_{i-1}, \vec{u}_{i-1} >} \right) \vec{u}_{i-1}$$

C.2. [15 points] Given $A = \begin{bmatrix} 1 & 2 & 5 & 2 \\ 0 & -1 & -2 & 1 \\ 3 & -6 & -9 & 18 \\ 0 & 1 & 2 & -1 \end{bmatrix}$ and $\vec{b} = \begin{bmatrix} 4 \\ -2 \\ -12 \\ 2 \end{bmatrix}$, solve $A\vec{x} = \vec{b}$. Express your solution set using column vector notation.

C.3. [15 points] The matrix $A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ has the property that there is at least one vector $\overrightarrow{x}$ for which $A\overrightarrow{x} = 5\overrightarrow{x}$. Find all such vectors.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Suppose $\{\overrightarrow{v}_1, \overrightarrow{v}_2, \overrightarrow{v}_3\}$ is a linearly independent set in $\mathbf{C}^9$. Determine if the set of vectors $\{2\overrightarrow{v}_1 + \overrightarrow{v}_2 + 3\overrightarrow{v}_3,\ \overrightarrow{v}_2 + 5\overrightarrow{v}_3,\ 3\overrightarrow{v}_1 + \overrightarrow{v}_2 + 2\overrightarrow{v}_3\}$ is linearly dependent or linearly independent.

M.2. [15 points] Prove **Theorem MIT: Matrix Inverse of a Transpose**. Specifically, prove the statement: Suppose A is an invertible matrix, then A^t is invertible.

M.3. [15 points] Do one of the following two problems

1. (a) Let $S = \{\vec{v}_1, \vec{v}_2\}$ be a set of non-zero vectors. Prove that S is linearly dependent, if and only if, one of the vectors in S is a scalar multiple of the other.

   (b) Prove the **DMAM Distributivity across Matrix Addition** vector space property of matrices. Specifically, prove the statment: If $\alpha \in \mathbf{C}$ and $A, B \in M_{mn}$,then $\alpha(A+B) = \alpha A + \alpha B$.

Do any two (2) of these "Other" problems

All of these problems can be done using matrix notation.

T.1. [15 points] Suppose that $\vec{x}$ and $\vec{y}$ are solution vectors to the non-homogeneous linear system of equations $LS\left(A, \vec{b}\right)$. Prove that $\vec{n} = \vec{x} - \vec{y}$ is a solution vector to the homogeneous system $LS\left(A, \vec{0}\right)$.

T.2. [15 points] Suppose A is a square matrix of size n satisfying $A^2 = AA = O_n$. Prove that the only vector $\vec{x}$ satisfying $(I_n - A)\vec{x} = \vec{0}$ is the zero vector. [Recall that O_n is the $n \times n$ zero matrix.]

T.3. [15 points] Suppose $A_{n\times m}$ and $B_{m\times n}$ are matrices such that $AB = I_n$. Let $\overrightarrow{b}$ be a particular vector in $\mathbf{C}^n$. Show that the system of equations $A\overrightarrow{x} = \overrightarrow{b}$ must be consistent. [Note that neither A nor B is square and so they cannot be invertible.]