Exam 2

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any two (2) of the following of these "Computational" problems

C.1. [8, 7 points] (Homework Problem M20 from section MO) Suppose $S = \{B_1, B_2, \cdots, B_p\}$ is a set of matrices from M_{mn}.Formulate appropriate definitions for the following terms and give an example of each.

1. (a) A linear combination of the elements of S.
   (b) A non-trivial relation of linear dependence on S.

C.2. [15 points] Given the linearly independent set $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \\ 0 \\ 1 \end{bmatrix} \right\}$, use the Gram-Schmidt Procedure to generate an orthogonal set $T = \{\vec{u}_1, \vec{u}_2, \vec{u}_3\}$ which has the same span as S.That is, $\langle S \rangle = \langle T \rangle$. [You may use: $\vec{u}_i = \vec{v}_i - \frac{\langle \vec{v}_i, \vec{u}_1 \rangle}{\langle \vec{u}_1, \vec{u}_1 \rangle}\vec{u}_1 - \frac{\langle \vec{v}_i, \vec{u}_2 \rangle}{\langle \vec{u}_2, \vec{u}_2 \rangle}\vec{u}_2 - \cdots - \frac{\langle \vec{v}_i, \vec{u}_{i-1} \rangle}{\langle \vec{u}_{i-1}, \vec{u}_{i-1} \rangle}\vec{u}_{i-1}$]

C.3. [8, 7 points] Find two linearly independent sets S, T whose spans equal the column space of matrix $B = \begin{bmatrix} 2 & 3 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ -1 & 2 & 3 & -4 \end{bmatrix}$.

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Prove property SMAM of matrices. Specifically, prove If $\alpha, \beta \in \mathbf{C}$ and $A \in M_{mn}$, then $\alpha(\beta A) = (\alpha\beta) A$.

M.2. [15 points] Homework Problem M25 of section MO. Let $U_{33} = \left\{ A \in M_{33} \mid [A]_{ij} = 0 \text{ whenever } i > j \right\}$. Find a set $R \subseteq M_{33}$ for which the span of R equals U_{33}, $\langle R \rangle = U_{33}$.

M.3. [15 points] Prove that if the set $S = \{\vec{v}_1, \vec{v}_2, \vec{v}_3, \cdots, \vec{v}_n\} \subseteq \mathbf{C}^m$ is linearly independent, then the set $T = \{\vec{v}_1 - \vec{v}_2,\ \vec{v}_2 - \vec{v}_3,\ \vec{v}_3 - \vec{v}_4, \cdots, \vec{v}_n - \vec{v}_1\}$ is linearly dependent.

Do any two (2) of these "Other" problems

T.1. [15 points] Use the method of mathematical induction to prove that if $A_1, A_2, \cdots, A_n$ are square matrices of the same size, then $(A_1A_2\cdots A_n)^t = A_n^t \cdots A_2^t A_1^t$ for all integers $n \geq 2$.

T.2. [15 points] Given a singular matrix $A \in M_{nn}$ and a linearly independent set $S = \{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n\}$ in $\mathbf{C}^n$, prove that it is not the case that the set $\{A\vec{v}_1, A\vec{v}_2, \cdots, A\vec{v}_n\}$ is also linearly independent. [Hint: consider the matrix B whose columns are the vectors in S.]

T.3. [15 points] In class we proved that if $A \in M_{mn}$ and $B \in M_{np}$ then the null space of B is contained in the null space of AB, $N(B) \subseteq N(AB)$. Prove that in the special case where A is non-singular, then $N(B) = N(AB)$.