Smith **Math 290** **Spring 2013**

Exam 1

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

You must do this problem.

D.1. [5 points] What objects have null spaces? Give the definition of **null space**.

D.2. [5 points] Give the definition of a **nonsingular matrix.**

Do both of these "Computational" problems

C.1. [15 points] Determine if the following system of equations is consistent and, if so, find the solution set.

$$\begin{cases} x_1 - x_2 - 3x_3 + x_4 + 2x_5 = 0 \\ -2x_1 + x_2 + 5x_3 \qquad\quad - 2x_5 = -4 \\ 4x_1 - 2x_2 - 10x_3 + x_4 + 5x_5 = 5 \end{cases}$$

C.2. [15 points] Suppose $[D \mid \vec{e}] = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 4 & 0 \\ 1 & -1 & 2 \end{bmatrix}$ is the result of the sequence of three row operations:

$\left[A \mid \vec{b}\right] \xrightarrow{R_2 \leftrightarrow R_4} [B \mid \vec{c}] \xrightarrow{3R_3} \left[C \mid \vec{d}\right] \xrightarrow{2R_1 + R_3} [D \mid \vec{e}]$. What is A?

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Suppose that $\mathbf{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix}$ are vectors in the null space of a matrix A with m rows and 4 columns. Prove that $\mathbf{t} = \begin{bmatrix} u_1 + v_1 \\ u_2 + v_2 \\ u_3 + v_3 \\ u_4 + v_4 \end{bmatrix}$ is a solution of $LS\left(A, \mathbf{0}\right)$ by explicitly showing that $\mathbf{t}$ solves the ith equation where i is any index satisfying $1 \leq i \leq m$.

M.2. [15 points] Prove Property DSAC of the vector space C^m. More precisely, prove: if $\alpha, \beta \in \mathbf{C}$ and $\mathbf{u} \in \mathbf{C}^m$, then $(\alpha + \beta)\,\mathbf{u} = \alpha\mathbf{u} + \beta\mathbf{u}$.

M.3. [8, 7 points] Prove Theorem NMRRI: A square matrix A is nonsingular if and only if the reduced row echelon form of A is the identity matrix.

Do any two (2) of these "Other" problems

T.1. [15 points] Prove that a system of linear equations is homogeneous **if and only if** it has the zero vector as a solution.

T.2. [15 points] Carefully explain why a linear system of equations $LS(A, \mathbf{b})$ is consistent **if and only if** the vector $\mathbf{b}$ is equal to a linear combination of the column vectors of A.

T.3. [15 points] Suppose that a certain system of n linear equations in k variables has a unique solution. Determine which if any of the following absolutely **must** be true? For each of the others, give an augmented matrix in reduced row-echelon form for a linear system illustrating why it is not the case that they absolutely must be true.

1. $n < k$
2. $n = k$
3. $n > k$
4. $n \leq k$
5. $n \geq k$