Exam 1

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Complete the following definitions

D.1. [3 points] Two systems of linear equations are **equivalent** if

D.2. [3 points] The **null space** of a matrix A, denoted $N(A)$ is

D.3. [4 points] The set of column vectors $S = \{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_n\}$ .is **linearly dependent** if

Do both of these "Computational" problems

C.1. [10, 5 points] By hand, solve the following system of linear equations. Write the solution set using column vector notation.

$$\begin{aligned} x_1 + 2x_2 - x_3 + x_4 &= 1 \\ x_2 + x_3 - x_4 &= 3 \\ -x_1 + x_2 + 7x_3 - x_4 &= 0 \end{aligned}$$

C.2. [15 points] Below are a matrix A and the matrix B that is row-equivalent to A and in reduced row-echelon form.

1. (a) Is A a singular matrix?
   (b) What are r, D, and F for matrix B?
   (c) Is the linear system of equations $LS(A, \mathbf{0})$ consistent? If so, how many solutions are there?
   (d) Are there any vectors $\mathbf{b}$ for which the linear system of equations $LS(A, \mathbf{b})$ is inconsistent?
   (e) Write the null space $N(A)$ of A as a span.

$$A = \begin{bmatrix} 1 & 3 & 1 & 1 & 1 & 1 & 5 \\ 2 & 6 & 1 & 4 & 1 & 2 & 7 \\ -3 & -9 & -1 & -7 & 1 & -1 & -3 \\ 1 & 3 & 1 & 1 & 3 & 3 & 11 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 0 & 3 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Suppose you are given a system of n linear equations in k variables. Determine which of the following is true. Briefly explain why it is true and why the others are false.

1. (a) both **b)** and **c)** below are correct
   (b) if the system has a unique solution then $n \geq k$
   (c) if $n = k$, then the system has a unique solution
   (d) neither **b)** nor **c)** is correct

M.2. [15 points] Property **DVAC** of column vectors is: if $\alpha \in \mathbf{C}$ and $\mathbf{u}, \mathbf{v} \in \mathbf{C}^m$, then $\alpha(\mathbf{u} + \mathbf{v}) = \alpha\mathbf{u} + \alpha\mathbf{v}$. Prove this property and write your proof in the style of the proof of Property **DSAC** given in the textbook.

M.3. [15 points] Prove the following half of Theorem **PSPHS.** Suppose that $\mathbf{w}$ is a solution to the linear system of equations $LS(A, \mathbf{b})$. If $\mathbf{y} = \mathbf{w} + \mathbf{z}$ for some vector $\mathbf{z} \in N(A)$, then $\mathbf{y}$ is a solution of $LS(A, \mathbf{b})$ Use the notation for system $LS(A, \mathbf{b})$ given below.

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \cdots + a_{1n}x_n &= b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \cdots + a_{2n}x_n &= b_2 \\ &\vdots \\ a_{m1}x_1 + a_{m2}x_2 + a_{m3}x_3 + \cdots + a_{mn}x_n &= b_m \end{aligned}$$

Do any two (2) of these "Other" problems

T.1. [10, 5 points] Given the set

$$S = \left\{ \begin{bmatrix} 5x_3 - 6x_5 \\ 2x_3 + x_5 \\ x_3 \\ x_5 \\ x_5 \end{bmatrix} : x_3,\ x_5 \in \mathbf{C} \right\}$$

1. Find a 4×5 matrix A, that is **not** in reduced row-echelon form, whose null space is the set S.
2. Write S as a span.

T.2. [15 points] Let $S = \{\vec{v}_1, \vec{v}_2\}$ be a set of vectors. Prove that S is linearly dependent, **if and only if** one of the vectors in S equals a scalar multiple of the other.

T.3. [15 points] Suppose that $W = \{\vec{w}_1, \vec{w}_2, \vec{w}_3, \cdots, \vec{w}_p\}$ is a set of vectors in $\mathbb{C}^{21}$ and that $\vec{u}$ and $\vec{v}$ are both in $\langle W \rangle$. Prove that $\vec{u} + \vec{v}$ is also in the span $\langle W \rangle$.