Exam 1

I affirm this work abides by the university's Academic Honesty Policy.

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Print Name, then Sign

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do both of these "Computational" problems

C.1. [15 points] Solve the following system of linear equations by hand. Write the solution set using column vector notation.

$$\begin{aligned} x_1 + 3x_2 + x_3 + 5x_4 &= 6 \\ 2x_1 + x_2 - 3x_3 &= 2 \\ x_2 + x_3 + 2x_4 &= 2 \end{aligned}$$

C.2. [15 points] Below are a matrix A and the matrix B in reduced row-echelon form that is row equivalent to A.

1. Is A a singular matrix?
2. What are r, D, and F for matrix B?
3. Is the linear system of equations $\mathcal{LS}(A, \mathbf{0})$ consistent? If so, how many solutions are there?
4. Are there any vectors $\mathbf{b} \in \mathbf{C}^7$ for which the linear system of equations $\mathcal{LS}(A, \mathbf{b})$ is inconsistent? Why?
5. What is the null space $\mathcal{N}(A)$ of A? (Write your answer in column vector form.)

$$A = \begin{bmatrix} 1 & 3 & 1 & 1 & 1 & 1 & 5 \\ 2 & 6 & 1 & 4 & 1 & 2 & 7 \\ -3 & -9 & -1 & -7 & 1 & -1 & -3 \\ 1 & 3 & 1 & 1 & 3 & 3 & 11 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 & 0 & 3 & 0 & 1 & 2 \\ 0 & 0 & 1 & -2 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Do any two (2) of these "In Class, Text, or Homework" problems

M.1. [15 points] Find a 4×5 matrix A, that is **not** in reduced row-echelon form, whose null space is the set

$$\left\{ x_2 \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 7 \end{bmatrix} + x_4 \begin{bmatrix} -6 \\ 0 \\ -5 \\ 1 \\ 1 \end{bmatrix} : x_2,\ x_4 \in \mathbf{C} \right\}$$

M.2. [15 points] Suppose that A is the coefficient matrix of a consistent linear system of equations and that two of the columns of A are identical. Prove that there must be an infinite number of solutions to the system of equations $LS\left(A,\vec{0}\right)$.

M.3. [15 points] Prove Theorem NSMRRI: A square matrix A is nonsingular if and only if the reduced row echelon form of A is the identity matrix.

Do the first and either of the remaining "Other" problems

T.1. [15 points] Consider the system of linear equations $LS\left(A,\vec{b}\right)$ where A and $\vec{b}$ are given below. If the augmented matrix $\left[A|\vec{b}\right]$ is row-reduced until the first 6 columns are in reduced row-echelon form we obtain the matrix B (also given below).

$$A=\begin{bmatrix} 0 & 0 & 0 & 1 & 2 & -1 \\ 1 & 2 & 0 & 0 & 1 & -1 \\ 1 & 2 & 2 & 0 & -1 & 1 \\ 1 & 2 & 2 & 1 & 1 & 0 \\ 2 & 4 & 4 & 3 & 4 & -1 \end{bmatrix}, \quad \vec{b}=\begin{bmatrix} a \\ b \\ c \\ d \\ e \end{bmatrix}, \quad B=\begin{bmatrix} 1 & 2 & 0 & 0 & 1 & -1 & b \\ 0 & 0 & 1 & 0 & -1 & 1 & -\frac{1}{2}b+\frac{3}{2}d-\frac{1}{2}e \\ 0 & 0 & 0 & 1 & 2 & -1 & -2d+e \\ 0 & 0 & 0 & 0 & 0 & 0 & a+2d-e \\ 0 & 0 & 0 & 0 & 0 & 0 & c-3d+e \end{bmatrix}$$

1. Briefly explain why the null space of the matrix $L=\begin{bmatrix} 1 & 0 & 0 & 2 & -1 \\ 0 & 0 & 1 & -3 & 1 \end{bmatrix}$ can tell us which vectors $\vec{b}$ make $LS\left(A,\vec{b}\right)$ consistent.
2. Find and express in column vector notation the set $T=\left\{\vec{b}\in\mathbf{C}^5 : LS\left(A,\vec{b}\right)\right.$ is a consistent system of equat
3. Choose a vector $\vec{b}_1$that is **not** in T and use your calculator to show that $LS\left(A,\vec{b}_1\right)$ has no solutions.

T.2. [15 points] Find a polynomial $f(x)=ax^3+bx^2+cx+d$ of degree 3 such that $f(1)=1,\ f(2)=5,\ f'(1)=2$, and $f'(2)=9$.

T.3. [15 points] Suppose that $\mathbf{u}=\begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v}=\begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ are solutions of the homogeneous system of linear equations $LS(A,\mathbf{0})$. Using Beezer's notation, prove that $\mathbf{t}=\mathbf{u}+\mathbf{v}$ is also a solution of $LS(A,\mathbf{0})$.