Mathematics 280 **Spring 2008**

Exam 4

April 17, 2008

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Technology used: ____________________________
Only write on one side of each page.

- Show all of your work. Calculators may be used for numerical calculations and answer checking only.

Problems

1. [15 points] Let f be a function on two variables that has continuous partial derivatives of all orders and consider the points $A(1,3)$, $B(3,4)$, $C(1,6)$ and $D(6,14)$. The directional derivative of f at A in the direction of the vector $\overrightarrow{AB}$ is $3\sqrt{5}$, and the directional derivative at A in the direction of $\overrightarrow{AC}$ is 25. Find the directional derivative at A in the direction of the vector $\overrightarrow{AD}$.

2. [15 points] Do **one** (1) of the following.

   (a) Find parametric equations for the line tangent to the curve of intersection of the surfaces $x^2+y^2=4$ and $x^2+y^2-z=0$ at the point $\left(\sqrt{2},\sqrt{2},4\right)$.

   (b) Around the point $(1,0)$ in the plane

      i. Is $f(x,y)=x^2(y+1)$ more sensitive to changes in x or to changes in y? Why?

      ii. What ratio of dx to dy will make df equal zero at $(1,0)$.

3. [15 points] Find the absolute maximum and minimum values, if they exist, of

$$f(x,y)=2x^3+y^4$$

   where the domain is the set $D=\{(x,y): y^2\le 1-x^2\}$.

4. [15 points] Do **one** (1) of the following using the method of Lagrange multipliers.

   (a) Find the dimensions of the rectangle of largest area that can be inscribed in the ellipse $x^2/9+y^2/25=1$ with sides parallel to the coordinate axes. What is the largest area?

   (b) Find the point(s) on the surface whose equation is $xyz=1$ closest to the origin. Although this set is unbounded, you may use the geometric fact that there **is** an absolute minimum value.

5. [15 points] Compute the average value of $f(x,y)=x\sin(xy)$ over the rectangle $R=[0,\pi/2]\times[0,1]$.

6. [15 points] Evaluate the double integral.

$$\int_0^3 \int_{\sqrt{x/3}}^1 e^{(y^3)}\, dy\, dx$$

7. [10 points] Set up iterated integral(s) for the volume of the solid that remains when a square hole of side length 2 is drilled through the center of a sphere of radius $\sqrt{2}$.