Mathematics 280A **Fall 2012** **Multivariate Calc.** **Exam 4**

____
Name

Directions

A. Only write on one side of each page and show all of your work.

B. Pay attention to problem statements. If you are asked to "Set up a definite/iterated integral or integrals ..." you do not need to evaluate the integral or integrals.

C. Each definite integral you set up should be expressed entirely in terms of one variable, each iterated double integral should be expressed entirely in terms of one pair of variables. You need not simplify algebra.

---

1. [4, 4 points] Briefly respond to both of the following.

   (a) Articulate a fundamental meaning of a line integral for a **scalar** field.

   (b) Articulate a fundamental meaning of a surface integral for a **vector** field.

2. [5 points] Find the area of the triangle with vertices $A\,(1,-2,3)$, $B\,(0,-3,5)$, and $C\,(2,-4,5)$.

   (a) [2 points] Extra Credit: What is an equation for the plane containing this triangle?

3. [15 points] Set up a definite integral or integrals equal to $\int_C f\ ds$ where $f\,(x,y) = x^3y$ and C is the portion of the polynomial $y = x^5$ for $x = 1$ to $x = 3$.

4. [15 points] Set up a definite integral or integrals to compute the length of a helix that wraps 10 times around the elliptical cylinder $\frac{x^2}{16} + \frac{y^2}{9} = 1$ which extends from $z = 0$ to $z = 5$. [Hint: the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is traced out exactly once by $x = a\cos(\theta)$ and $y = b\sin(\theta)$ as θ goes from 0 to 2π.]

5. [5, 8, 3 points] Consider the vector field $\vec{F} = (2 + 2xy^2)\,\hat{\imath} + \left(\frac{2}{z^2} + 2x^2y\right)\hat{\jmath} + \left(4 - \frac{2y}{z^3}\right)\hat{\mathbf{k}}$ for $z > 0$.

   (a) Use the component test to show that $\vec{F}$ is conservative.

   (b) Find a potential function for $\vec{F}$.

   (c) Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where C is a curve that starts at $(1,1,1)$ and ends at $(2,2,2)$.

6. [10 points] Do **one** (1) of the following.

   (a) Use the Divergence Theorem (see below) to evaluate $\iint\limits_S \vec{F} \cdot d\vec{A}$ where $\vec{F} = (z-x)\,\hat{\imath} + (x-y)\,\hat{\jmath} + (y-z)\,\hat{\mathbf{k}}$ and S is the sphere of radius 4 centered at the origin with $d\vec{A}$ oriented inward.

   Divergence Theorem: Let $\vec{F}$ be a vector field whose components have continuous partial derivatives, and let S be a piecewise smooth, oriented, closed surface that encloses the solid region D. Then $\iint\limits_S \vec{F} \cdot d\vec{A} = \iiint\limits_D \left(\text{div } \vec{F}\right)\ dV$

   (b) Determine an expression in x, y, z Cartesian coordinates for the infinitesimal area element $d\vec{A}$ for the surface $z = y^2 - x^2$.

7. [15 points] Charge is distributed on the surface of a right circular cylinder of radius R and height H so that the area charge density is proportional to the distance from the plane containing the base of the cylinder. Compute the total charge on the cylinder in terms of R, H, and the maximum charge density σ_0.

8. $[3, 2, 8, 3$ points$]$ Do all of the following

   (a) Use the grid below to sketch the vector field $\vec{F} = y\,\hat{\imath} + 2\,\hat{\jmath}$ for the region with $-2 \le x \le 2$ and $-2 \le y \le 2$.Plot an output for each of the points provided **on the grid below**.

   (b) Sketch the curve $y = x^2 - 2$ for $x = -2$ to $x = 2$ on the grid below.

   (c) Compute $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}$ is the function in part a. and C is the curve in part b.

   (d) Use your plot to explain why your answer in part c. is reasonable.

Extra Grid