Mathematics $280A, 280B$ **Fall 2012**

Exam 1

September 13, 2012

_____
Name

Technology used: _____

Only write on one side of each page.
Show all of your work.
Calculators may be used for numerical calculations and answer checking only.

1. [10 points] State the definition of the **partial derivative of** $f(x, y)$ **with respect to** x at the point (x_0, y_0).

2. [15 points] Use the distance formula to obtain an equation (in a standard form) for the set of points $P(x, y, z)$ that are equidistant from the points $P_1(1, 2, 3)$ and $P_2(-1, 0, 0)$. What geometric surface is the shape of this set of points?

3. [15 points] Write an equation in slopes-intercept form for the plane that passes through the three points $(2, 1, 3)$, $(4, 5, 3)$, $(-1, 1, 6)$.

4. [9, 6 points] Display the values of the function $f(x, y) = 1 - |x|$ in two ways:

   (a) by carefully drawing an assortment of at least four level curves in the function's domain and labeling each level curve with its function value and

   (b) by sketching the surface $z = f(x, y)$.

5. [15 points] Find the following limit by first rewriting the fraction

$$\lim_{(x,y)\longrightarrow(2,-4)} \frac{y+4}{x^2y - xy + 4x^2 - 4x}$$

6. [15 points] By considering different paths of approach, show that the function given below has no limit as $(x, y, z) \longrightarrow (0, 0, 0)$.

$$f(x, y, z) = \frac{xy + xz + yz}{x^2 + y^2 + z^2}$$

7. [15 points] The **one-dimensional wave equation** in physics displayed below is used to model wave motions where w is the wave height, x is the distance variable, t is the time variable, and c is the velocity with which the waves are propagated. Show that the function $w = f(x, t) = \sin(x + ct)$ is a solution of the wave equation.

$$\textbf{Wave equation}: \quad \frac{\partial^2 w}{\partial t^2} = c^2 \frac{\partial^2 w}{\partial x^2}$$