Mathematics 232-A Exam 4 Spring 2006

April 20, 2006

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Name

Technology used: ______________________

Directions: Only write on one side of each page. Use terminology correctly. Partial credit is awarded for correct approaches so justify your steps.
You are to turn in **six** problems.

Do any two (2) of these computational problems

C.1. Do **both** of the following.

(a) Suppose $\overrightarrow{v}$ is an eigenvector of the $n \times n$ matrix A with associated eigenvalue 3. Show that $\overrightarrow{v}$ is also an eigenvector for the matrix $A^2 + 4I_n$. What is the associated eigenvalue?

(b) Show that $\begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$ is an eigenvector for the matrix $\begin{bmatrix} 2 & -6 & 6 \\ 1 & 9 & -6 \\ -2 & 16 & -13 \end{bmatrix}$ and determine the corresponding eigenvalue.

C.2. The characteristic polynomial of $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ is $\lambda^2 (\lambda - 1)^2$. Find the the geometric and algebraic multiplicities of each eigenvalue and determine a basis for each eigenspace. Do **NOT** use technology.

C.3. Show the following function $T : P_2 \to \mathbf{C}^3$ is a linear transformation.

$$T(f) = \begin{bmatrix} f(0) \\ f'(1) \\ f(2) \end{bmatrix}$$

C.4. Do one of the following.

(a) Find a basis for the kernel of the linear transformation $T : P_2 \to \mathbf{C}^3$ given by

$$T(f) = \begin{bmatrix} f(0) \\ f'(1) \\ f(2) \end{bmatrix}.$$

(b) Find a basis for the range of the linear transformation $T : P_2 \to \mathbf{C}^3$ given by

$$T(f) = \begin{bmatrix} f(0) \\ f'(1) \\ f(2) \end{bmatrix}.$$

Do two (2) of these problems from the text, homework, or class.

M.1. Prove Theorem CILTI, Composition of Injective Linear Transformations is Injective.

Suppose that $T : U \to V$ and $S : V \to W$ are injective linear transformations. Then $(S \circ T) : U \to W$ is an injective linear transformation.

M.2. Prove Theorem CSLTS, Composition of Surjective Linear Transformations is Surjective.

Suppose that $T : U \to V$ and $S : V \to W$ are surjective linear transformations. Then $(S \circ T) : U \to W$ is a surjective linear transformation.

M.3. Prove Theorem KILT, Kernel of an Injective Linear Transformation.

Suppose that $T : U \to V$ is a linear transformation. Then T is injective **if and only if** the kernel of T is trivial, $K(T) = \{\mathbf{0}\}$.

M.4. Suppose that A and B are similar matrices. Prove that A^5 and B^5 are similar matrices.

Do two (2) of these problems you've not seen before.

T.1. Prove that a function $T : V \to W$ is a linear transformation **if and only if** for all scalars $c, d \in \mathbf{C}$ and for all vectors $\overrightarrow{v}, \overrightarrow{w} \in V$ we have $T(c\overrightarrow{v} + d\overrightarrow{w}) = c\,T(\overrightarrow{v}) + d\,T(\overrightarrow{w})$.

T.2. An $n \times n$ matrix A is **nilpotent** if, for some positive integer k, $A^k = O$, where O denotes the $n \times n$ zero matrix. Prove that if A is nilpotent, then A is not invertible.

T.3. An $n \times n$ matrix A is called **nilpotent** if, for some positive integer k, $A^k = O$, where O is the $n \times n$ zero matrix. Prove that 0 is the only eigenvalue of any nilpotent matrix.

T.4. Suppose that A is a 4×4 matrix with exactly two distinct eigenvalues, 5 and -9 and let $E_A(5)$ and $E_A(-9)$ be the corresponding eigenspaces, respectively. Write all possible characteristic polynomials of A, in factored form, that are consistent with $\dim(E_A(-9)) = 2$.