Mathematics 232-A Exam 3 Spring 2006

March 30, 2006

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Name

Technology used: ______________________

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any three (3) of these computational problems

C.1. Option: Find the inverse of the following matrix **by hand**. You may not use a calculator.

$$\begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 2 & 2 \\ 1 & 1 & 3 & 3 & 3 \\ 1 & 1 & 1 & 4 & 4 \\ 1 & 1 & 1 & 1 & 5 \end{bmatrix}$$

C.2. Write $A = \begin{bmatrix} 0 & 2 \\ 1 & 3 \end{bmatrix}$ as a product of elementary matrices.

C.3. Two matrices A and B commute if $AB = BA$. Show that the set of matrices in M_{22} that commute with $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ is a subspace of M_{22} and find a basis for that subspace.

C.4. Let $S = \{\vec{v}_1, \vec{v}_2, \cdots, \vec{v}_p\}$ be a collection of vectors in a vector space V. Show that the span of S, $< S >$ is a subspace of V and that $\dim(V) \leq p$.

Do one (1) of these problems from the text, homework, or class.

You may NOT just cite a theorem or result in the text. You must prove these results.

M.1. Suppose $S = \{\vec{v}_1, \cdots, \vec{v}_t\}$ is a basis for a vector space V and $\vec{w} \neq \vec{0}$ is a vector in the span of S, $< S >$. Prove there is a basis, T, of V where $\vec{w} \in T$.

M.2. Suppose A is an invertible matrix of size n. Prove that $\overline{(A^{-1})} = \left(\overline{A}\right)^{-1}$.

Do one (1) of these problems you've not seen before.

T.1. Let V be a vector space and U and V subspaces of W. Show that the set of vectors $U + V = \{\vec{u} + \vec{v} \in W : \vec{u} \in U \text{ and } \vec{v} \in V\}$ is a subspace of W.

T.2. If A is an invertible matrix of size n, prove $\det\left(A^{-1}\right) = \frac{1}{\det(A)}$.

Do this mathematical induction problem

Induct Use mathematical induction to prove the following.

Let A be a square matrix of size $n \geq 2$ and B the matrix obtained after multiplying each entry of row i of A by the nonzero constant α (a type 2 elementary row operation). Use the technique of mathematical induction to prove that $\det(A) = \frac{1}{\alpha}\det(B)$.