Mathematics 232-A Exam 2 Spring 2006

February 23, 2006

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Name

Technology used: ______________________

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any three (3) of these computational problems

C.1. Is the set of vectors $S = \{\mathbf{w}_1, \mathbf{w}_2, \mathbf{w}_3, \mathbf{w}_4\} \left\{ \begin{bmatrix} 7 \\ 3 \\ 5 \\ 4 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 5 \\ 3 \\ 9 \\ 7 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 3 \\ 8 \\ 6 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \\ 7 \\ 3 \\ 1 \end{bmatrix} \right\}$ linearly dependent or linearly independent? If it is linearly dependent, first write one of the $\mathbf{w}$'s as a linear combination of the others and then write the set T that is a subset of S, is linearly independent, and for which $< T > = < S >$.

C.2. Write each of the following complex numbers in the form $a + bi$.

(a) $i\,(3 - 2i) + 7\left(\overline{-2 + i}\right)$.

(b) $(4 - 2i)\,(-3 + i)$

(c) $\frac{2-i}{3+4i}$

C.3. Consider the following vectors in $\mathbf{C}^4$.

$$\vec{v}_1 = \begin{bmatrix} 1/2 \\ 1/2 \\ 1/2 \\ 1/2 \end{bmatrix}, \quad \vec{v}_2 = \begin{bmatrix} 1/2 \\ 1/2 \\ -1/2 \\ -1/2 \end{bmatrix}, \quad \vec{v}_3 = \begin{bmatrix} 1/2 \\ -1/2 \\ 1/2 \\ -1/2 \end{bmatrix}$$

Find all vectors $\vec{v}_4$ in R^4 so that $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ form an orthonormal set.

Although you don't need it, the formula for the Gram-Schmidt process is

$$\vec{u}_i = \vec{v}_i - \left(\frac{< \vec{v}_i, \vec{u}_1 >}{< \vec{u}_1, \vec{u}_1 >} \right) \vec{u}_1 - \cdots - \left(\frac{< \vec{v}_i, \vec{u}_{i-1} >}{< \vec{u}_{i-1}, \vec{u}_{i-1} >} \right) \vec{u}_{i-1}$$

C.4. The matrix $A = \begin{bmatrix} 4 & -2 \\ -1 & 3 \end{bmatrix}$ has the property that there is at least one vector $\vec{x}$ for which $A\vec{x} = 5\vec{x}$. Find all such vectors.

Do any two (2) of these problems from the text, homework, or class.

You may NOT just cite a theorem or result in the text. You must prove these results.

M.1. Prove that if the matrix A is nonsingular and B is any appropriately sized matrix, then $N(AB) \subseteq N(B)$.

M.2. Prove DMAM (Distributivity across Matrix Addition): If $\alpha \in \mathbf{C}$, and $A, B \in M_{mn}$, then $\alpha(A+B) = \alpha A + \alpha B$.

M.3. Prove if $\{w_1, w_2, w_3\}$ is a linearly dependent set in $\mathbf{C}^{23}$, then the set

$$\{2w_1 + w_2 + 3w_3, -3w_1 + 2w_2 + 4w_3, w_1 + 2w_2 + 3w_3\}$$

is linearly dependent.

Do one (1) of these problems you've not seen before.

T.1. Suppose $A_{n\times m}$ and $B_{m\times n}$ are matrices such that $AB = I_n$. Let $\overrightarrow{b}$ be a particular vector in R^n. Show that the system of equations $A\overrightarrow{x} = \overrightarrow{b}$ must be consistent.

T.2. Use the Principle of Mathematical Induction to prove that the statement $P(n)$ given by $\sum_{k=1}^{n}(2k-1) = n^2$ holds for all positive integers.