Mathematics 232-A Exam 1 Spring 2006

February 2, 2006

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Name

Technology used: ______________________

Directions:

- **Only write on one side of each page.**
- **Use terminology correctly.**
- **Partial credit is awarded for correct approaches so justify your steps.**

Do any six (6) of the following problems.

1. Solve the following system of linear equations by hand. Write the solution set using column vector notation.

$$\begin{aligned} x_1 + 2x_2 + 5x_3 + 2x_4 &= 4 \\ -x_2 - 2x_3 + x_4 &= -2 \\ 3x_1 - 6x_2 - 9x_3 + 18x_4 &= -12 \\ x_2 + 2x_3 - x_4 &= 2 \end{aligned}$$

2. Find a polynomial $f(x)$ of degree 3 such that $f(1) = 1$, $f(2) = 5$, $f'(1) = 2$, and $f'(2) = 9$.

3. If
$$[A|\mathbf{b}] \xrightarrow{R_i \longleftrightarrow R_j} [B|\mathbf{c}] \xrightarrow{\beta R_i} [C|\mathbf{d}] \xrightarrow{\alpha R_i + R_j} [D|\mathbf{e}]$$
are a sequence of row operations that convert the augmented matrix $[A|\mathbf{b}]$ to $[D|\mathbf{e}]$. Write down a sequence of row operations that will convert $[D|\mathbf{e}]$ to $[A|\mathbf{b}]$.

4. Given linear system $LS(A, \mathbf{b})$ with solution set S, and linear system $LS(B, \mathbf{c})$ with solution set T, that is the result of the following row operation on the augmented matrices: $[A|\mathbf{b}] \xrightarrow{\alpha R_i + R_j} [B|\mathbf{c}]$. Show $S \subseteq T$.

5. Prove Theorem NSRRI: Suppose that A is a square matrix and B is a row-equivalent matrix in reduced row-echelon form. Then A is nonsingular if and only if B is the identity matrix.

6. Below are a matrix A and the matrix B in reduced row-echelon form that is row equivalent to A.
   (a) Is A a singular matrix?
   (b) What are r, D, and F for matrix B?
   (c) Is the linear system of equations $LS(A, \mathbf{0})$ consistent? If so, how many solutions are there?
   (d) If $\mathbf{b}$ is not the zero vector, $\mathbf{0}$, is the linear system of equations $LS(A, \mathbf{b})$ consistent? If so, how many solutions are there?

(e) What is the null space $N(A)$ of A? (Write your answer in column vector form.)

$$A = \begin{bmatrix} 1 & 2 & 1 & 3 & 1 & 0 & 5 \\ 2 & 4 & 1 & 4 & 1 & 0 & 7 \\ -3 & -6 & -1 & -5 & 1 & -2 & -3 \\ 1 & 2 & 1 & 3 & 3 & -2 & 11 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 0 & 1 & 0 & 0 & 2 \\ 0 & 0 & 1 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

7. Suppose that α is any constant and that $\mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_n \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} v_1 \\ \vdots \\ v_n \end{bmatrix}$ are solutions of the homogeneous system of linear equations $LS(A, \mathbf{0})$. Prove that $\mathbf{t} = \begin{bmatrix} \alpha u_1 \\ \vdots \\ \alpha u_n \end{bmatrix}$ is also a solution of $LS(A, \mathbf{0})$. [Be sure to explicitly show that $\mathbf{v}$ solves the system of equations.]