Mathematics 221 Fall 1998 Exam 4

December 2, 1998

______________ **Name**

Technology used: ______________________________

Textbook/Notes used: ______________________________

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. **Only write on one side of each page.**

The Problems

1. Find all maxima, minima and saddle points of
$$f(x, y) = 9x^3 + \frac{y^3}{3} - 4xy.$$
2. Find the absolute maximum and minimum of the function
$$f(x, y) = x^2 - xy + y^2 + 1$$
on the closed triangular plate in the first quadrant bounded by the lines $x = 0,\ y = 4,\ y = x.$
3. Find the points on the ellipse $x^2 + 2y^2 = 1$ where $f(x, y) = xy$ has its extreme values.
4. Sketch the region of integration and then evaluate
$$\int_1^{\ln 8} \int_0^{\ln y} e^{x+y}\, dx\, dy$$
5. Find the volume of the region that lies under the paraboloid $z = x^2 + y^2$ and above the triangle enclosed by the lines $y = x,\ x = 0,$ and $x + y = 2$ in the xy-plane.
6. Find the volume of the solid in the first octant bounded by the coordinate planes, the plane $x = 3$ and the parabolix cylinder $z = 4 - y^2$.
7. Find the moment of inertia about the y-axis of a thin plate (lamina) bounded by the line $y = 1$ and the parabola $y = x^2$ if the density is $\rho(x, y) = y + 1$.
8. Use double integrals to find the volume of a sphere of radius R.
9. A flat circular plate has the shape of the region $x^2 + y^2 \leq 1$. The plate, including the boundary, is heated so the temperature at the point (x, y) is
$$T(x, y) = x^2 + 2y^2 - x.$$
Find the temperatures at the hottest and coldest points on the plate.

10. Rewrite (but do not evaluate) the triple integral

$$\int_0^1 \int_{-1}^0 \int_0^{y^2} dz\, dy\, dx$$

in the order

(a) $dy\, dz\, dx$

(b) $dx\, dz\, dy$

11. Find the average value of $F(x, y, z) = x^2 + y^2 + z^2$ over the cube in the first octant bounded by the coordinate planes and the planes $x = 1$, $y = 1$, and $z = 1$.

12. Use Lagrange multipliers to find the point on the surface $z = xy + 1$ that is nearest the origin.

13. If x thousand dollars is spent on labor and y thousand dollars is spent on equipment, the output of a certain factory may be modeled by

$$Q(x, y) = 60x^{1/3}y^{2/3}.$$

units. Assume $\$120,000$ is available.

How should money be allocated between labor and equipment to generate the largest possible output?

14. In the above problem, use the Lagrange multiplier λ and differentials to estimate the change in the maximum output of the factory that would result if the money available for labor and equipment is increased by $\$1,000$.

15. Over what region in the xy-plane does

$$\iint_R \left(4 - x^2 - 2y^2\right) dA$$

have its maximum value?

16. Find the area enclosed by one leaf of the rose $r = 12\cos(3\theta)$.

17. Find the volume of the region in the first octant bounded by the coordinate planes and the planes $x + z = 1$, and $y + 2z = 2$.