Mathematics 221 Fall 1998 Exam 1

September 17

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Name

Technology used: ____

Textbook/Notes used: ____

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Only write on one side of each page.

The Problems

1. The vectors $\overrightarrow{a}$, $\overrightarrow{b}$, $\overrightarrow{c}$ in the figure all lie in a plane. By arranging the vectors head to tail sketch (a) $\overrightarrow{a} - \overrightarrow{b}$, (b) $\overrightarrow{a} - \overrightarrow{b} + \overrightarrow{c}$, and (c) $\overrightarrow{a} - 2\overrightarrow{b}$.

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2. a
   (a) Find a unit vector that is tangent to the curve $y = x^2$ at the point $P(2,4)$.
   (b) Find a unit vector that is normal (orthogonal) to the curve $y = x^2$ at the point $P(2,4)$.
3. Find a unit vector that is tangent to the curve $3x^2 + 8xy + 2y^2 - 3 = 0$ at the point $(1,0)$.
4. Find a unit vector that is tangent to the curve $y = \int_0^x \sqrt{3 + t^4}\, dt$ at the point $(0,0)$.
5. Write $\overrightarrow{a} = 5\,\mathbf{i} + 12\,\mathbf{j}$ as a product of a scalar and a unit vector in the same direction as $\overrightarrow{a}$.

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6. Describe the set of points in space whose coordinates satisfy the given inequality and equation.

$$x^2 + y^2 \leq 1, \quad z = 3.$$

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7. The points A, B, C are the corners of a triangle.
   (a) Find the vector represented by the directed line segment from C to the midpoint M of side AB.
   (b) Find the vector represented by the segment from C to the point that lies two-thirds of the way from C to M on the median CM.

8. Let $ABCD$ be a quadrilateral in space. Show that the two segments joining the midpoints of opposite sides of $ABCD$ bisect each other. (Hint: Show the segments have the same midpoint.)

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9. Given $\overrightarrow{a} =< \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{6}} >$, and $\overrightarrow{b} =< 0, \frac{1}{\sqrt{2}}, -1 >$ find

   (a) The scalar component (scalar projection) of $\overrightarrow{b}$ in the direction of $\overrightarrow{a}$.

   (b) The vector projection of $\overrightarrow{b}$ in the direction of $\overrightarrow{a}$.

10. Write $\overrightarrow{b} =< 8, 4, -12 >$ as the sum of a vector parallel to $\overrightarrow{a} =< 1, 2, -1 >$ and a vector orthogonal to $\overrightarrow{a}$.

11. This problem shows that cancellation does not hold for the dot product. Find an example of vectors $\overrightarrow{a}, \overrightarrow{b_1}, \overrightarrow{b_2}$ where $\overrightarrow{a} \neq \overrightarrow{0}$, $\overrightarrow{a} \cdot \overrightarrow{b_1} = \overrightarrow{a} \cdot \overrightarrow{b_2}$, but $\overrightarrow{b_1} \neq \overrightarrow{b_2}$.

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12. Find, and label, two of the interior angles of the triangle ABC whose vertices are $A(-1, 0, 2)$, $B(2, 1, -$ and $C(1, -2, 2)$.

13. Find the angle between the diagonal of a cube and the diagonal of one of its faces.

14. Find the angle between the diagonal of a cube and one of the edges it meets at a vertex.

15. Find the angle between the tangent lines to the two curves $y = x^3$, and $y = \sqrt{x}$ at the point $Q(1, 1)$.

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16. Find a unit vector in the direction of $\overrightarrow{a} \times \overrightarrow{b}$ if $\overrightarrow{a} =< -8, -2, -4 >$, and $\overrightarrow{b} =< 2, 2, 1 >$.

   (a) Find a vector $\overrightarrow{n}$ perpendicular to the plane of the points $P(1, -1, 2)$, $Q(2, 0, -1)$, and $R(0, 2, 1)$.

   (b) Find the area of triangle PQR.

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17. Given vectors $\overrightarrow{a}$, $\overrightarrow{b}$, and $\overrightarrow{c}$, use the dot product and cross product, as appropriate, to write the formulas for the following (read carefully):

   (a) The vector projection of $\overrightarrow{a}$ onto $\overrightarrow{b}$.

   (b) A vector orthogonal to $\overrightarrow{a}$ and $\overrightarrow{b}$.

   (c) A vector with the length of $\overrightarrow{a}$ and the direction of $\overrightarrow{b}$.

   (d) A vector orthogonal to $\overrightarrow{a} \times \overrightarrow{b}$ and $\overrightarrow{c}$.

18. This problem shows that cancellation does not hold for the cross product. Find an example of vectors $\overrightarrow{a}, \overrightarrow{b_1}, \overrightarrow{b_2}$ where $\overrightarrow{a} \neq \overrightarrow{0}$, $\overrightarrow{a} \times \overrightarrow{b_1} = \overrightarrow{a} \times \overrightarrow{b_2}$, but $\overrightarrow{b_1} \neq \overrightarrow{b_2}$.

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19. Find parametric and symmetric equations for the line through the point $P(3,-4,-1)$ and parallel to the vector $\mathbf{i}+\mathbf{j}+\mathbf{k}$.

20. Find a parametric form for the line through the point $P\,(2,3,0)$ and perpendicular to the vectors $\overrightarrow{a} = \mathbf{i}+2\,\mathbf{j}+3\,\mathbf{k}$ and $\overrightarrow{b} = 3\,\mathbf{i}+4\,\mathbf{j}+5\,\mathbf{k}$.

    (a) Give an example, in parametric form, of two lines in space that are **skew**.

    (b) Give an example, in symmetric form, of two lines in space that are **parallel**.

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21. Find the distance from the point $P\,(2,1,3)$ to the line $x = 2+2t,\ y = 1+6t,\ z = -3-5t$.

22. Does the line $x = 1-t,\ y = 3t,\ z = 1+t$ have any points in common with the sphere $2x^2+2y^2+2z^2+x+y+z = 9$? If so, find all such points.