Mathematics 221 **Fall 2003**

Exam 4

November 25, 2003

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Name

Technology used: ____

Directions: Be sure to include in-line citations every time you use technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

I. (20 points each) Do any two (2) of the following.

1. The **average value** of a function $f(x, y)$ over a region D is defined to be

$$f_{\text{ave}} = \frac{1}{A(D)} \iint\limits_{D} f(x, y)\, dA$$

   where $A(D)$ is the area of region D. Compute the average value of $f(x, y) = x \sin(xy)$ over the rectangle $R = [0, \pi/2] \times [0, 1]$.

2. Integrate the function $f(x, y) = x/y$ over the region in the first quadrant bounded by the lines $y = x, y = 2x, x = 1, x = 2$.

3. Find the value of the improper integral $I = \int_0^\infty e^{-x^2}\, dx$ by using polar coordinates to compute

$$I^2 = \left(\int_0^\infty e^{-x^2}\, dx\right)\left(\int_0^\infty e^{-y^2}\, dy\right) = \int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dy\, dx.$$

II. (15 points) Set up an iterated double integral that describes the **surface area** of the frustum of the cone $z = 4\sqrt{x^2 + y^2}$ between the planes $z = 1$ and $z = 4$. **Do not evaluate the integral.**

1. (over)

III. (15 points) Do any two (2) of the following

1. **Guaranteed Problem from Worksheet:** Use iterated integrals in polar, cylindrical or spherical coordinates to express the the volume inside the sphere $x^2 + y^2 + z^2 = 25$ and outside the cylinder $(x-1)^2 + y^2 = 1$.**(Do not evaluate.)**
2. Set up, (but do not evaluate) the triple integral **in spherical coordinates** which represents the volume of the solid above the plane $z = 1$ and interior to both the cone $z = \sqrt{x^2 + y^2}$ and the sphere $x^2 + y^2 + z^2 = 4$.
3. The following iterated triple integral is being integrated over a solid domain D.

$$\int_{-1}^{1} \int_{x^2}^{1} \int_{0}^{1-y} 1\, dz\ dy\ dx$$

   (a) Explain why the projection of D into the xz - plane is the region bounded by the x - axis and the parabola $z - 1 = -x^2$.

   (b) Rewrite (but do not evaluate) the integral as an equivalent iterated integral in the order $dy\, dz\, dx$.

IV. (15 points) Do one (1) of the following.

1. The change of variables

$$\langle x, y \rangle = T(u, v) = \langle 3u + 2v,\ u + v \rangle$$

   transforms the square $S = \{(u, v) : 0 \le u \le 1,\ 0 \le v \le 1\}$ in the uv-plane into a corresponding region D in the xy-plane.

   (a) Sketch the region D in the xy-plane.

   (b) Compute the Jacobian of the transformation.

2. Let D be the region in bounded by the lines $x - y = 0$, $x - y = 1$, $x + y = 1$, and $x + y = 2$. Evaluate

$$\iint\limits_{D} \frac{x + y}{\cos(x - y)}\, dA.$$

   [Hint: Make an appropriate change of variables.]