

November 4, 2003

 Name

Technology used: _____

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

I. (15 points) Write out the Chain Rule formula for $\frac{\partial z}{\partial w}$ for the case where $z = f(x, y)$, $x = x(s, t)$, $y = y(s, t)$, $s = s(u, v, w)$, $t = t(u, v, w)$.

II. (15 points) Do one (1) of the following.

- For both of the following, find the limit, if it exists, or show the limit does not exist. Do **not** use $\varepsilon - \delta$ techniques to find limits that exist.

(a)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^3 + xy^2}{x^2 + y^2}$$

(b)

$$\lim_{(x,y) \rightarrow (0,0)} \frac{(x+y)^2}{x^2 + y^2}.$$

- Use Clairaut's Theorem to show that if the third order partial derivatives of f are continuous then

$$f_{xxy} = f_{yxy} = f_{yyx}.$$

III. (20 points) Do one (1) of the following.

- Given the surface $z = f(x, y)$ with implicit equation $x^2 - 2y^2 - 3z^2 + xyz = 4$, find the equation of the tangent plane at the point $(3, -2, -1)$.
- Four positive numbers, each less than or equal to 50 are rounded to the first decimal place and then multiplied together. Use differentials to estimate the maximum error in the computed product that might result from the rounding.

IV. (20 points) Find and classify the local maximum, local minimum, and saddle points of

$$f(x, y) = x^3 - 3xy + y^3.$$

[There is more than one such point.]

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V. (15 points each) Do any two (2) of the following.

1. Find the point on the plane $2x - y + z = 1$ that is closest to the point $(-4, 1, 3)$.
2. Find the absolute maximum and minimum values, if they exist, of

$$f(x, y) = 2x^3 + y^4$$

where the domain is the set $D = \{(x, y) : x^2 + y^2 \leq 1\}$.

3. Use Lagrange multipliers to find the maximum value of the function $z = f(x, y) = x^4 + y^4 + z^4$ subject to the constraint $x^2 + y^2 + z^2 = 1$. Are you sure there really is a maximum?