Mathematics 221 Fall 2003

Exam 2

October 7

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Name

Technology used: ______________________

Directions: Cite any use of technology. For partial credit, if you are unsure of your answer to a problem be sure to describe what you do know and where you think your error is. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

I (15 points each) Do any two (2) of the following problems.

1. Find the time or times (values of t) in the interval $t \geq 0$ when $\overrightarrow{F}(t) = (\sin t)\ \mathbf{i} + (t-2)\ \mathbf{j} + (\cos t)\ \mathbf{k}$, and its derivative are orthogonal.
2. Find the point of intersection of $\overrightarrow{F}_1(t) = \ < 2t, 2t-1, 3+4t^2 >$ and $\overrightarrow{F}_2(s) = \ < 6-s, 2s-7, s^2-9 >$ and compute the angle between the tangent vectors at this point.
3. A certain helix, when parametrized by arc length has the parametrization
$$\overrightarrow{F}(s) = \left\langle \cos\left(\frac{s}{\sqrt{2}}\right), \sin\left(\frac{s}{\sqrt{2}}\right), \frac{s}{\sqrt{2}} \right\rangle,\ s \geq 0.$$
Find the curvature of this helix at the point where $s = 1$.

II (20 points each) Do and two (2) of the following problems.

1. The paraboloid $2y = (x-1)^2 + z^2$ and the plane $x + z = 1$ intersect along a curve in $\mathbf{R}^3$. Find a parametrization $\overrightarrow{F}(t)$ for this curve.
2. Find the position function of a particle that starts at $\widehat{\mathbf{j}}$ with an initial velocity of $\widehat{\mathbf{i}} + \widehat{\mathbf{k}}$ and has acceleration $\overrightarrow{\mathbf{A}}(t) = t\ \widehat{\mathbf{i}} + t^2\ \widehat{\mathbf{j}} + \cos(2t)\ \widehat{\mathbf{k}}$.
3. Find the unit tangent, principal unit normal and unit binormal vectors for the circular helix
$$\vec{\mathbf{r}}(t) = \cos(t)\ \widehat{\mathbf{i}} + \sin(t)\ \widehat{\mathbf{j}} + t\ \widehat{\mathbf{k}}.$$

III (15 points each) Do any two (2) of the following problems.

1. If $\overrightarrow{G}(t) \neq \overrightarrow{0}$, show that
$$\frac{d}{dt}\left[\left\|\overrightarrow{G}(t)\right\|\right] = \frac{1}{\left\|\overrightarrow{G}(t)\right\|}\left(\overrightarrow{G}(t) \cdot \overrightarrow{G}\,'(t)\right).$$
[Hint: $\left\|\overrightarrow{G}(t)\right\|^2 = \overrightarrow{G}(t) \cdot \overrightarrow{G}(t)$.]
2. Suppose $\overrightarrow{R}_1(t) = \langle x(t), y(t), z(t) \rangle,\ a \leq t \leq b$ and $\overrightarrow{R}_2(u) = \overrightarrow{R}_1(g(u)),\ \alpha \leq u \leq \beta$ are two parametrizations of a curve C where $t = g(u)$, $a = g(\alpha)$, $b = g(\beta)$, and $g'(u) > 0$ for all values of u. Show the arc length of the curve C is independent of parametrization by showing that $\int_a^b \left\|\overrightarrow{R}_1'(t)\right\| dt = \int_\alpha^\beta \left\|\overrightarrow{R}_2'(u)\right\| du$. [Hint: Use substitution on the first integral just as you did on the "independence of parametrization" problem on the last project.]

3. The DNA molecule has the shape of double helix. The radius of each helix is about 10 angstroms and each helix rises about 34 angstroms during each turn for a total of about 2.9×10^8 complete turns. Compute the length of each helix.