Mathematics 221 Fall 1998 Exam 3

November 4, 1998

Technology used: ______________________________

Textbook/Notes used: ______________________________

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. **Only write on one side of each page.**

The Problems

1. Sketch (reasonably carefully) two level surfaces of $f(x,y,z) = x^2 + z^2$.

2. The ideal gas law says that $PV = kT$, where P is the pressure of a confined gas, V its volume, T its temperature (in degrees Kelvin), and k is a constant. Express P as a function of V and T and describe the level curves of this function.

3. Show that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{x-y}{\sqrt{x^2+y^2}}.$$

4. If $f(x,y) = x^5 y^7 + x\cos(y^2)$ find

   (a) $f_x(x,y)$

   (b) $f_y(x,y)$

   (c) $\frac{\partial^2 f}{\partial x \partial y}(x,y)$.

5. An equation that models the concentration, C, of a substance as it diffuses through a region over time is (where x indicates position, t is time and δ is a constant)

$$\frac{\partial C}{\partial t} = \delta \frac{\partial^2 C}{\partial x^2}.$$

   Show that the following function satisfies the above diffusion equation.

$$C(x,t) = t^{-1/2} e^{-x^2(4\delta t)}.$$

6. Find $\frac{\partial w}{\partial r}$ if

$$w = \frac{x+y}{2-z},\ x = 2rs,\ y = \sin(rt),\ z = st^2.$$

7. Write out the formula for $\frac{\partial u}{\partial r}$ if $u = f(x,y,z,w), x = x(s,t,r), y = y(s,t,r), z = z(s,t,r)$ and $w = w(s,t,r)$.

8. A pail attached to a rope 2 meters long is swung at the rate of 1 revolution per second. Find the tangential and normal components of the acceleration. Assume the rope is swung in a level plane.

9. Let f have continuous partial derivatives and suppose the maximal directional derivative of f at $P_0\,(1,2)$ has magnitude 50 and is attained in the direction from P_0 toward $Q\,(3,-4)$. Use this information to find $\nabla f\,(1,2)$.

10. Find the slope of the tangent line to the level curve at the point $P\,(1,1,3)$ on the surface $z = x^2 + xy^2 + y^3$.

11. Suppose the position vector of a moving particle is given by $\overrightarrow{r}\,(t)$. Show the tangential component of the object's acceleration can be computed by the formula

$$a_{\overrightarrow{T}} = \frac{\overrightarrow{r}\,'(t) \cdot \overrightarrow{r}\,''(t)}{|\overrightarrow{r}\,'(t)|}.$$

12. Use differentials to show that near $(0,0)$

$$\frac{1}{1+x-y} \approx 1 - x + y.$$

13. According to Poiseuille's Law, the resistance to the flow of blood offered by a cylindrical blood vessel of radius r and length x is

$$R\,(r,x) = \frac{cx}{r^4}$$

for a constant $c > 0$. A certain blood vessel in the body is 8 cm long and has a radius of 2 mm. Estimate (using differentials) the percentage change in R when x is increased by 3% and r is decreased by 2%.