Mathematics 213 Spring 2004 Exam 2

March 11, 2004

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Name

Directions: Only write on one side of each page.

Useful Information

1. **Hilbert's parallel axiom**: If l is any line and P is any point not incident with l then there is at most one line through P parallel to l.
2. **Statement** 4.7: If a line, distinct from two parallel lines, intersects one of the two parallel lines, then it also intersects the other.
3. **Statement** 4.12: If $m \| l$ and $l \| n$, then $m \| n$.

The Problems

I. Do all three of the following.

1. Using any previous results, prove part (d) of Proposition 3.13.
   If $AB < CD$ and $CD < EF$, then $AB < EF$.
2. Using any previous results and the existence of angle bisectors, prove the following portion of Proposition 4.4
   The bisector of an angle is unique.
3. Using any previous result, prove Proposition 4.7.
   (Statement 4.7) $\Longleftrightarrow$ (Hilbert's parallel axiom)

II. Do any two (2) of the following.

1. Using any results from Chapter 3 prove the following.
   Given angle $\measuredangle ABC$ and ray $\overrightarrow{BD}$ opposite to ray $\overrightarrow{BC}$, if point E is on the same side of line $\overleftrightarrow{BC}$ as A, then exactly one of the following is true.
   (a) E is interior to angle $\measuredangle ABC$
   (b) E is on ray $\overrightarrow{BA}$
   (c) E is interior to angle $\measuredangle ABD$.
2. Let γ be a circle with center O, and let A and B be two points on γ. The segment AB is called a **chord** of γ. Suppose segment AB is not a diameter of γ and let M be the midpoint of segment AB (so $M \neq O$). Prove that line $\overleftrightarrow{OM}$ is perpendicular to line $\overleftrightarrow{AB}$.
3. Using any result through Proposition 4.11, prove

$$\text{(Statement } S.12) \Longrightarrow \text{(Hilbert's parallel property).}$$

   You do **not** need to prove the other half of the equivalence

$$\text{(Hilbert's parallel property)} \Longleftrightarrow \text{(Statement } S.12).$$