Honors 213 Spring 2005 Exam 1

Feb 8, 2005

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Name

Directions: Only write on one side of each page.

I. Do all three (3) of the following

1. Prove the following theorem about Scorpling Flugs. The Axioms and pertinent definitions are in the "Useful Information" section of this examination.

   Theorem 4: There cannot be two or more pushy flugs.

2. Prove the following logical statement (modus ponens) is a tautology.

$$((H \Rightarrow C) \wedge H) \Longrightarrow C$$

3. Prove the following proposition of Incidence geometry. The axioms and previous propositions are in the "Useful Information" section of this examination.

   Proposition 2.4: For every point there is at least one line not passing through it.

II. Do any two (2) of the following

1. Prove the following proposition of Incidence geometry.

   Proposition 2.6 For every point P there are at least two distinct points neither of which is P.

2. Here is an interpretation of the undefined terms of incidence geometry: Fix a circle in the Euclidean plane. Each and every Euclidean point interior to the circle is interpreted to be a "point". Each and every chord of the circle (a chord is the portion of a Euclidean line that is interior to the circle) is interpreted to be a "line". Interpret "incidence" of a "point" with a "line" to mean that the point lies on the chord in the usual Euclidean sense.

   (a) Which of the axioms of Incidence geometry are satisfied by this interpretation? Explain.

   (b) This interpretation has a parallel property. Is it the elliptic, Euclidean, or hyperbolic parallel property? Explain.

3. Explain why, in Incidence geometry it is impossible to either prove or disprove the statement "for every line l and every line m not equal to l, l and m are incident with exactly the same number of points".

Useful Information

Scorpling Flugs Axioms and Definitions

1. Given two distinct flugs, either the first scorples the second or the second scorples the first (the possibility of both is not excluded).

2. No flug scorples itself.

3. If A, B and C are flugs (not necessarily distinct), such that A scorples B and B scorples C , then A scorples C.

4. There are exactly four distinct flugs.

Definitions

1. A flug that scorples every other flug is called a **pushy** flug.

2. A flug that is scorpled by every other flug is called a **passive** flug.

Incidence Geometry

Undefined terms: point, line, incident with

Incidence Axiom 1

For every point P and for every point Q not equal to P there exists a unique line l incident with P and Q.

Incidence Axiom 2

For every line l there exist at least two distinct points incident with l.

Incidence Axiom 3.

There exist three distinct points with the property that no line is incident with all three of them.

Proposition 1 (2.1) *If l and m are distinct lines that are not parallel, then l and m have a unique point in common.*

Proposition 2 (2.2) *There exist three distinct lines that are not concurrent.*

Proposition 3 (2.3) *For every line there is at least one point not lying on it.*

Proposition 4 (2.4) *For every point there is at least one line not passing through it.*

Proposition 5 (2.5) *For every point P there exist at least two lines through P.*