Mathematics 213 Spring 2004 Exam 1

Directions: Only write on one side of each page.

The Problems

I. Do both of the following.

1. Use a truth table to prove the following logical statement is a tautology.

$$\tilde{}\,[H \Rightarrow C] \Longleftrightarrow H \wedge \tilde{}C$$

2. Using the incidence axioms and any previous result, formally prove Proposition 2.5. For every point P there are at least two distinct lines passing through it. Use a two column proof.

II. Do any three (3) of the following.

1. Using any result through Proposition 2.5 of incidence geometry prove the following.
   For every point P there are at least two distinct points, neither of which is P. [This was Proposition 2.6 on the graded homework set.]
2. Recall that a projective plane is a model of incidence geometry satisfying the elliptic parallel property and in which every line has at least three points incident with it.
   Let M be a projective plane and let M' be the interpretation of the undefined terms obtained by interpreting M' points to be the lines of M, the M' lines to be the points of M and incidence in M' the same as incidence in M.
   Formally prove that M' satisfies Incidence Axiom 1.
3. Call the geometry obtained from Incidence Geometry by removing Incidence Axiom 3 "Partial Geometry".
   (a) Explain what it means to say that Incidence Axiom 3 is **independent** of Partial Geometry.
   (b) Use models to show that Incidence Axiom 3 cannot be proven using the axioms of Partial Geometry.
4. Show that it is not the case that any two models of Incidence Geometry with exactly four points must be isomorphic.