Honors 213 - A **Spring 2002**

Exam 1

February 19, 2001

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Name

Technology used: ______________________________

Textbook/Notes used: ______________________________

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. **Do any two (2) of the following.**

   (a) Using any result from chapter 2 (but none of the exercises from that chapter), prove that if M is any projective plane in which each line is incident with exactly $n+1$ points then each point is incident with at least $n+1$ lines.

   (b) Using any previous results (including the first claim of Proposition 3.3), prove the second claim of Proposition 3.3.

   Given $A * B * C$ and $A * C * D$, then $A * B * D$.

   (c) Negate the following well-formed-statement (here $l \,||\, m$ is defined to mean that line l is parallel to line m.)

   $$\forall l \,\forall m \,\forall n \ [(l \,||\, m \ \& \ m \,||\, n) \Rightarrow (l \,||\, n)]$$

2. **Do any three (3) of the following.**

   (a) Using any previous result, prove Proposition 2.4. For every point there is at least one line not passing through it.

   (b) Using any result from chapter 2, show that if M is a projective plane in which every line is incident with exactly $n+1$ points then every point is incident with no more than $n+1$ lines. (You may assume the previous result, proven in class, that every point of M is incident with at least $n+1$ lines.)

   (c) Using any result through Proposition 3.2 and the "Same Side Lemma", prove the "Opposite Side Lemma"

   If $P * Q * R$ and l is a line, distinct from $\overleftrightarrow{PQ}$ that meets $\overleftrightarrow{PQ}$ at Q, then P and R are on opposite sides of l.

   (d) Recall that a well-formed-statement (wfs) is **independent** of an axiomatic system if neither that statement nor its negation can be deduced from the axioms. Prove the following (wfs) is independent of the axioms of incidence geometry.

   "For any two lines l and m there exists a one-to-one correspondence between the set of points incident with line l and the set of points incident with line m."