Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. Only write on one side of each page.

## Problems

1. ( 20 points) Using any previous results, prove part ( $b$ ) of Proposition 3.21. Given angles $\measuredangle P, \measuredangle Q, \measuredangle R$. If $\measuredangle P<\measuredangle Q$ and $\measuredangle Q \cong \measuredangle R$, then $\measuredangle P<\measuredangle R$.
2. ( 20 points) Using any previous results, prove the following portion of Proposition 4.3.Every segment has a midpoint (do NOT show the midpoint is unique).
3. ( 20 points each) Do any three (3) of the following.
(a) In the following interpretation, all incidence axioms and the first two betweenness axioms hold. Explain why betweenness axiom 3 fails. Use the usual Euclidean model except interpret the betweenness relation $A * B * C$ to mean " $B$ is the midpoint of segment $A C$ ".
(b) In the figure on the blackboard, the pairs of angles ( $\left.\measuredangle A^{\prime} B^{\prime} B^{\prime \prime}, \measuredangle A B B^{\prime \prime}\right)$ and ( $\left.\measuredangle C^{\prime} B^{\prime} B^{\prime \prime}, \measuredangle C B B^{\prime \prime}\right)$ are called pairs of corresponding angles cut off on $l$ and $l^{\prime}$ by transversal $t$.Prove corresponding angles are congruent if and only if alternate interior angles are congruent.
(c) Using any result through Chapter 4 prove the following. Let $\gamma$ be a circle with center $O$, and let $A$ and $B$ be two points on $\gamma$. The segment $A B$ is called a chord of $\gamma$. Let $M$ be the midpoint of segment $A B$. Prove that if $O \neq M$, then $\overleftrightarrow{O M}$ is perpendicular to $\overleftrightarrow{A B}$.
(d) Using any result through the corollaries to Theorem 4.3, prove the following. If $A * B * C$ and $\overleftrightarrow{D C} \perp \overleftrightarrow{A C}$ then $A D>B D>C D$. (See the figure on the board.)
