## Textbook/Notes used:

Directions: Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. Only write on one side of each page.

## The Problems

## Do any two (2) of the following.

1. Using any previous results, prove part $(d)$ of Proposition 3.13.

If $A B<C D$ and $C D<E F$, then $A B<E F$.
2. Let $\gamma$ be a circle with center $O$, and let $A$ and $B$ be two points on $\gamma$. The segment $A B$ is called a chord of $\gamma$. Suppose segment $A B$ is not a diameter of $\gamma$ and let $M$ be the midpoint of segment $A B$ (so $M \neq O$ ). Prove that line $\overleftrightarrow{O M}$ is perpendicular to line $\overleftrightarrow{A B}$.
3. Using any previous result, prove the portion of Proposition 4.9

$$
\left(\text { Statement } S_{4.9}\right) \Rightarrow \text { (Hilbert's parallel postulate) }
$$

Here statement $S_{4.9}$ is: "If $t$ is a transversal to lines $l$ and $m, l \| m$, and $t \perp l$, then $t \perp m$."

## Do any two (2) of the following.

1. Using any result up to and including Proposition 4.5 and exercise 26 prove the following. If $A * B * C$ and line $\overleftrightarrow{D C}$ is perpendicular to line $\overleftrightarrow{A C}$, then $A D>B D>C D$. [Hint: Use Proposition 4.5.]
2. Using any previous result, prove Proposition 4.11

Hilbert's parallel postulate $\Rightarrow$ the angle sum of every triangle is exactly $180^{\circ}$.
3. Definition: Let $l$ and $l^{\prime}$ be two distinct lines and $t$ a transversal meeting $l, l^{\prime}$ at $B$ and $B^{\prime}$, respectively. Let $A, C$ be on $l$ with $A * B * C$ and $A^{\prime}, C^{\prime}$ on $l^{\prime}$ with $A, A^{\prime}$ on the same side of $t$ and $A^{\prime} * B^{\prime} * C^{\prime}$. Further let $B^{\prime \prime}$ on $t$ be such that $B * B^{\prime} * B^{\prime \prime}$. Then the pairs of angles ( $\left.\measuredangle A^{\prime} B^{\prime} B^{\prime \prime}, \measuredangle A B B^{\prime \prime}\right)$ and $\left(\measuredangle C^{\prime} B^{\prime} B^{\prime \prime}, \measuredangle C B B^{\prime \prime}\right)$ are called corresponding angles cut off on $l$ and $l^{\prime}$ by transversal $t$.

Using any result in Chapter 4 (but not any exercises) prove that corresponding angles are congruent if and only if alternate interior angles are congruent.
4. Using any result in Chapter 4 previous to exercise 31 prove that if line $l$ meets circle $\gamma$ at two points $C$ and $D$ and $C * P * D$, then $P$ is interior to $\gamma$.[That is, show that if $O$ is the center of $\gamma$ then $O P<O C$.]

