Mathematics 122 **Fall 2002**

Exam 2

October 8, 2002

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Name

Technology used:____ Directions:

Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. $(4, 4, 7$ points$)$

   (a) Evaluate $\int \frac{1}{x+5}\ dx$

   (b) Evaluate $\int (x + 1/2)e^{x^2+x}\ dx$

   (c) What is the average value of the volumes of all possible spheres with radii between 3 and 6? [Useful fact: The volume of a sphere of radius r is $4/3\pi r^3$.]

2. $(10, 5$ points$)$

   (a) Find the general solution to the following first-order, nonlinear, separable differential equation with initial condition.
   $$\frac{dy}{dx} = \frac{x^2}{y}\frac{\sqrt{y^2+5}}{\sqrt{x^3+1}}$$

   (b) Find the particular solution corresponding to the initial condition $y(2) = 2$.

3. (10 points) Find the area of region bounded by the curve $y = 2x^2+1$ and the lines $x = -2$, $y = 4x+1$.

4. $(10, 10$ points$)$ The mass density of oil, measured in kilograms per square meter, in a circular oil slick on the surface of the ocean at a distance r meters from the center of the slick is given by $\rho(r) = \frac{50}{1+r}$ $\frac{\text{kg}}{\text{m}^2}$.

   (a) Suppose the slick extends from $r = 0$ to $r = 10,000$ m. Using a partition with n subintervals and either the left or right endpoints, write a Riemann sum approximating the total mass of oil in the slick. [Mass is measured in kilograms (kg).]

   (b) Write the definite integral that is equal to the limit as $||P|| \to 0$ of your Riemann sum. Do not evaluate this definite integral.

5. Do **one** of the following.

   (a) (20 points) A function f has the following properties. For all $a \le x \le b$, $f(x) < 0$, $f'(x) < 0$, and $f''(x) > 0$.
   Which of the numerical estimates (left endpoint, right endpoint, midpoint, trapezoid or Simpson's rules) for $\int_a^b f(x)\ dx$ will always produce an overestimate? Which will always produce an underestimate? And for which is there not enough information to determine the relationship of the estimate to $\int_a^b f(x)\ dx$?

(b) $(5, 5, 10$ points) Suppose $f(x)$ is a monotone increasing function on the interval $[a, b]$. Briefly explain why each of the following is true.

i. For any integer n, the right endpoint approximation R_n is an overestimate of the value of $\int_a^b f(x)\, dx$.

ii. For any integer n, the left endpoint approximation L_n is an underestimate of the value of $\int_a^b f(x)\, dx$.

iii. If we use the average, $\frac{R_n+L_n}{2}$, of R_n and L_n as an estimate for $\int_a^b f(x)\, dx$, then the error in our approximation can be no worse than $\frac{R_n-L_n}{2}$. That is,

$$\left| \int_a^b f(x)\, dx - \frac{R_n + L_n}{2} \right| \leq \frac{R_n - L_n}{2}.$$

6. (20 points) Do one of the following

(a) Find the derivative $H'(x)$ of

$$H(x) = x^2 \int_5^{\cos(x)} \frac{\ln(t)}{t^4 + 7}\, dt.$$

(b) Find a function f that satisfies the equation

$$\tan(x) + e^x = \int_5^x \sqrt{f(t) - 2}\, dt.$$

Hint: take the derivative of both sides.