Mathematics 122 – A **Fall 2002**

Exam 1

September 17, 2002

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Name

Technology used:____ Directions:

Be sure to include in-line citations, including page numbers if appropriate, every time you use a text or notes or technology. Include a careful sketch of any graph obtained by technology in solving a problem. **Only write on one side of each page.**

The Problems

1. (7 points each) Evaluate the following indefinite integrals.

   (a)
   $$\int \left(2e^x + 4\sec(x)\tan(x)\right)\, dx$$

   (b)
   $$\int \frac{1}{t^2}\left(\frac{2}{t} - \frac{5}{t^3}\right)\, dt$$

   (c)
   $$\int \frac{x^3 - \sqrt[3]{x} + 1}{x} dx$$

   (d)
   $$\int \frac{23}{\sqrt{1-x^2}} + e^x\, dx$$

2. (16, 3 points)

   (a) Find a formula $a(k)$, $k = 0, 1, 2, \cdots$ that gives the following sequence. Express your answer using the 'bar' ($k^{\underline{n}}$) notation.

   $$3, -3, 1, 33, 111, 253, 477, 801, 1243, 1821, 2553, \cdots$$

   Show all of your work.

   (b) Express your formula in the 'nonbar' (k^n) notation but **DO NOT SIMPLIFY** your answer.

3. (20 points) Use the summation techniques from our discussion of sequences and/or Section 5.2 of our textbook to find the exact area bounded by the graph of $y = x^2 + 2x$,the x- axis, and the vertical lines $x = 0$ and $x = 3$.

4. Do **one** of the following

(a) (20 points) Suppose that $a(k)$, $k = 0, 1, 2, \cdots$ is an arbitrary (but unknown) sequence and that $A(k)$, $k = 0, 1, 2, \cdots$ is one of the discrete antiderivatives of $a(k)$ but we don't know whether or not $A(0) = 0$. Show that

$$A(n+1) - A(0) = \sum_{k=0}^{n} a(k) = a(0) + a(1) + a(2) + \cdots + a(n).$$

(b) (20 points) The following limit gives the exact area of a region in the plane. Carefully describe that region. **DO NOT EVALUATE** the limit.

$$\lim_{n\to+\infty} \sum_{k=1}^{n} \left[3\left(2 + \frac{4k}{n}\right)^3 + \left(2 + \frac{4k}{n}\right)^2 + 5 \right] \frac{4}{n}.$$

5. (20 points) An airplane has a constant acceleration while moving down the runway from rest. What is the acceleration of the plane at liftoff if the plane requires 900 feet of runway before lifting off at $88\frac{\text{ft}}{\text{s}}$?

Useful Facts

- $$\sum_{k=1}^{n} 1 = n \qquad \sum_{k=1}^{n} k = \frac{n(n+1)}{2}$$
  $$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{k=1}^{n} k^3 = \frac{n^2(n+1)^2}{4}$$
- $k^{\underline{n}} = k(k-1)(k-2)\cdots(k-n+1)$
- $D_k\left[k^{\underline{n}}\right] = nk^{\underline{n-1}}$ and If $a(k) = k^{\underline{n}}$, then $A(k) = \frac{1}{n+1}k^{\underline{n+1}} + C$
- $D_k\left[2^k\right] = 2^k$ and if $a(k) = 2^k$, then $A(k) = 2^k + C$
- $D_k\left[r^k\right] = (r-1)r^k$ and if $a(k) = r^k$ then $A(k) = \frac{1}{r-1}r^k + C$
$k^0 = k^{\underline{0}}$	Discrete Antiderivative $\to$	$k^{\underline{1}} = k + C$
$k^1 = k^{\underline{1}}$	Discrete Antiderivative $\to$	$\frac{1}{2}k^{\underline{2}} = \frac{1}{2}k(k-1) + C$
$k^2 = k^{\underline{2}} + k^{\underline{1}}$	Discrete Antiderivative $\to$	$\frac{1}{3}k^{\underline{3}} + \frac{1}{2}k^{\underline{2}} = \frac{1}{6}k(2k-1)(k-1)$
$k^3 = k^{\underline{3}} + 3k^{\underline{2}} + k^{\underline{1}}$	Discrete Antiderivative $\to$	$\frac{1}{4}k^{\underline{4}} + k^{\underline{3}} + \frac{1}{2}k^{\underline{2}} = \frac{1}{4}k^2(k-1)^2$
$k^4 = k^{\underline{4}} + 6k^{\underline{3}} + 7k^{\underline{2}} + k^{\underline{1}}$	Discrete Antiderivative $\to$	$\frac{1}{5}k^5 - \frac{1}{2}k^4 + \frac{1}{3}k^3 - \frac{1}{30}k$